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A strategy to calibrate errors of Earth gravity models

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ABSTRACT

In this paper, three independent Earth gravity models (EGMs) of GO_CONS_GCF_2_TIM_R4, AIUB-GRACE03S and ULux_CHAMP2013s are combined to degree and order 120. The geoid models of these EGMs are computed and compared with the Global Positioning System (GPS) and levelling data over Fennoscandia. We found that the simple mean of these geoid models is closer to the GPS/levelling data than their weighted mean. This means that errors of the EGMs are not properly estimated as they are used in the weighted mean solution. We develop a method based on solving a nonlinear condition adjustment model to calibrate the errors so that the result of weighted mean becomes the same as that of the simple mean. Numerical results show slight changes in the errors of GRACE03S but large ones in those of GO_CONS_GCF_2_TIM_R4 and ULux_CHAMP2013s. Furthermore, the weighted mean solution considering the calibrated errors and some additional constraints is better than GOCO03S to degree and order 120 over Fennoscandia.

1. Introduction

The Challenging Mini-satellite Payload (CHAMP) (Reigber and Schwintzer, 2000), the Gravity Recovery and Climate Experiment (GRACE) (Tapley et al., 2005) and the Gravity Field and Steady-state Ocean Circulation Explorer (GOCE) (ESA, 1999) are three recent satellite gravimetry missions. In the CHAMP mission the high–low satellite-to-satellite tracking (SST) data between the CHAMP and global positioning system (GPS) satellites were analysed and different versions of the Earth's gravity models (EGMs) were presented based on such analysis. In the GRACE mission twin-satellites, more or less in the same orbit, follow each other and the distance between them is measured continuously. This type of measurements is called low–low SST and by combing them and the high–low SST data to the GPS satellites the orbit of the GRACE satellites are precisely determined and analysed for recovering EGMs. In the GOCE mission a new measurement technique is used which is so-called satellite gravity gradiometry. In fact GOCE observations are second-order derivatives of the Earth gravitational potential and recovering higher frequencies of the gravity field are expected from this mission. The orbit of GOCE is determined based on high–low SST data to the GPS satellites as well. For a good overview of analysing the GOCE data see Pail et al. (2011). Since the quality of long wavelength structure of the GOCE EGM is not very good, they are combined with the GRACE data which resulted to the combined GOCE–GRACE EGMs, for example GOCO01S (Pail et al., 2010), GOCO02S (Goiginger et al., 2011) and GOCO03S (Mayer-Gürr et al., 2012) were computed based on this strategy.

So far various EGMs have been presented based on these three missions and their combinations. Here we refer the readers to http://icgem.gfz-potsdam.de/ICGEM/modelstab.html to see a list of them. Today, the concentrations of the geodetic researchers are on the evaluation of the GOCE data. Hirt et al. (2011) compared some of GOCE EGMs with terrestrial gravimetric data over Switzerland, Austria and astrogeodetic deflections over Europe. They observed some improvements between degrees 160–165 and 180–185. Gruber et al. (2011) compared some of the GOCE EGMs for reproducing the orbit of GRACE and concluded that they do not outperform the GRACE orbit; therefore, combination of GRACE and GOCE data is useful. They found significant improvements between degrees 50 and 200 for geoid computation goal. Janak and Pitonak (2011) evaluated the GOCE products in central Europe and Slovakia. They mentioned that TIM2 and SPW2 to degree 210 are much better than the previous releases to the same degree and GOCO02S has a significant improvement comparing to GOCO01S. Sprlak et al. (2012) did the same study in Norway and mentioned that the direct solutions are highly affected by a priori information and time-wise solution is more reliable. Abdalla et al. (2012) evaluated the GOCE EGMs in Sudan and concluded that the SPW1, SPW2, TIM1, TIM2 and GOCO01S are consistent with the local data. Abdalla and Tenzer (2012) validated EGMs in New Zealand. Guimaraes et al. (2012) tested the EGMs in Brazil and found out that TIM3 is much better than the previous ones, as expected. Eshagh and Ebadi (2013) also investigated different EGMs and evaluated them over Fennoscandia. So far evaluation of the GOCE EGMs was done based on comparison of the EGM products with external sources of data, like gravity anomaly, disturbing gravity, geoid and/or astrogeodetic deflections. However, it should be considered that the errors of EGMs have been also presented. Wanger and McAdoo (2012)

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noticed that the errors of GOCE EGMs are not realistic and tried to calibrate them based on EGM08 (Pavlis et al., 2008, 2012). Eshagh (2013) studied the reliability and calibration of GOCE models as well.

In this paper we select and combine GO_CONS_GCF_2_TIM_R4 (TM4) model (Pail et al., 2011) delivered from analysis of the GOCE data, AIUB-GRACE03S (GRC) (Jäggi et al., 2011) computed from the GRACE data, ULux_CHAMP2013s (CMP) (Weigelt et al., 2013) which is the new EGM of derived from the CHAMP. In order to do better comparison we will compare the result of our combined EGM with GOCO03S (GOC). Here, we compare the simple and weighted mean values of these three EGMs and compare their geoid models to the Global Positioning System (GPS) and levelling data over territory of Fennoscandia. Furthermore, a new strategy is presented for calibration of the errors of EGMs based on a nonlinear condition adjustment model. This strategy will be used for computing a new combined EGM from CMP, GRC and TM4 to degree and order 120 and it is implementable once the new EGMs of the mentioned missions are available.

2. Spherical harmonic expression of geoid

Let the following be the well-known spherical harmonic expansion of the geoid (Heiskanen and Moritz, 1967, p. 88):

$$N = \frac{GM}{R\gamma}\sum_{n=2}^{L} k_n \left(\frac{R}{r}\right)^{n+1} \sum_{m=-n}^{n} t_{nm} Y_{nm}(\theta,\lambda) \tag{1a}$$

where GM is the geocentric gravitational constant, R the semi-major axis of the reference ellipsoid, r the geocentric distance of the points at which the geoid is computed, γ the normal gravity, t_{nm} the spherical harmonic coefficients of the disturbing potential, and $Y_{nm}(\theta, \lambda)$ the fully-normalised spherical harmonics of degree n and order m at a point with co-latitude and longitude of θ and λ. L stands for the maximum degree of the EGM. In order to somehow consider the errors of EGMs we can use the following coefficient which is in fact the Wiener filter:

$$k_n = \frac{c_n}{c_n + dc_n} \tag{1b}$$

where c_n and dc_n are respectively, signal and error degree variances of the EGM with the following expressions:

$$c_n = \sum_{m=-n}^{n} t_{nm}^2 \quad \text{and} \quad dc_n = \sum_{m=-n}^{n} \delta t_{nm}^2 \tag{1c}$$

and δt_{nm} stands for the error of t_{nm}.

The quality of the spherical harmonic coefficients of the Earth gravity field is deteriorated by increasing the degree of the field and k_n will try to filter the high degree coefficients based on their presented errors. It will not have any significant impact on the result if the spectral errors (dc_n) are considerably smaller than the signal (c_n). Nevertheless, judging about the quality of the spherical harmonic coefficients and their errors is not straightforward unless the resulted geoid models or gravity anomalies are compared to external sources of information like GPS/levelling data or terrestrial gravity anomalies.

3. Comparison of Earth gravity models

Comparison of one existing EGM with another can be a simple way to see how different they are. In statistical point of view, the mean value of EGMs should be closer to the true EGM in the presence of no systematic error. Simple and weight means can be considered as simple estimators for combining EGMs. However in order to see how far their results are from the true EGM they should be compared with the external and independent source of information. The problem which is discussed now is related to the case where the simple mean (SMN) delivers better results than the weighted mean (WMN). Therefore, we can conclude that the presented errors for the EGMs are not realistic. Finding the incorrect weights of the coefficients of each EGM is not straightforward. Nevertheless, these weights can be adjusted at least to get the same result as that of the SMN. To do so, a condition equation should be organised in such a way that it equates the WMN to the SMN.

Mathematically this idea can be written in the following form:

$$\sum_{i=1}^{N} t_{nm}^{i} \frac{p_{nm}^{i}}{\sum_{i=1}^{N} p_{nm}^{i}} = \frac{1}{N}\sum_{i=1}^{N} t_{nm}^{i} \tag{2a}$$

where i stands for the index number of each EGM and N is the total number of them, t_{nm}^{i} is the spherical harmonic coefficient of $_i$th EGM and $p_{nm}^{i} = 1/(\sigma_{nm}^{i})^2$ is the weight of the coefficients in which σ_{nm}^{i} is the error of each coefficient.

The unknowns of Eq. (2a) are p_{nm}^{i} which should be estimated but Eq. (2a) is nonlinear with respect to them the equation should be linearised:

$$\sum_{i=1}^{N} t_{nm}^{i} \frac{p_{nm}^{i,0}}{\sum_{i=1}^{N} p_{nm}^{i,0}} + \frac{\partial}{\partial p_{lk}^{i}} \sum_{i=1}^{N} t_{nm}^{i} \frac{p_{nm}^{i,0}}{\sum_{i=1}^{N} p_{nm}^{i,0}} = \frac{1}{N}\sum_{i=1}^{N} t_{nm}^{i} \tag{2b}$$

$$\left(\sum_{i=1}^{N} p_{nm}^{i,0}\right)^{-2} \sum_{k=1}^{N} t_{nm}^{k} \left(\delta_{ki}\sum_{i=1}^{N} p_{nm}^{i,0} - p_{nm}^{k,0}\right) = \frac{1}{N}\sum_{i=1}^{N} t_{nm}^{i} - \sum_{i=1}^{N} t_{nm}^{i} \frac{p_{nm}^{i,0}}{\sum_{i=1}^{N} p_{nm}^{i,0}} \tag{2c}$$

$p_{nm}^{i,0}$ stands for the initial weights and in the matrix form we have

$$\mathbf{B}\Delta\mathbf{p} = \Delta l \tag{2d}$$

where $\mathbf{B}$ is the coefficient vector of the weights, $\Delta\mathbf{p}$ vector of corrections to the weights and Δl is the mis-closure value or the difference between the SMN and WMN solutions. $\mathbf{B}$ and $\Delta\mathbf{p}$ have the following structures:

$$\mathbf{B} = \left(\sum_{i=1}^{N} p_{nm}^{i,0}\right)^{-2} \left[t_{nm}^{1}\left(\sum_{i=1}^{N} p_{nm}^{i,0} - p_{nm}^{1,0}\right) - \sum_{\substack{i=1 \\ i\neq 1}}^{N} \left(p_{nm}^{i,0} t_{nm}^{i}\right) t_{nm}^{2} \left(\sum_{i=1}^{N} p_{nm}^{i,0} - p_{nm}^{2,0}\right) - \sum_{\substack{i=1 \\ i\neq 2}}^{N} \left(p_{nm}^{i,0} t_{nm}^{i}\right) \ldots \right] \tag{2e}$$

$$\Delta\mathbf{p} = \begin{bmatrix} \Delta p_{nm}^{1} & \Delta p_{nm}^{2} & \cdots & \Delta p_{nm}^{N} \end{bmatrix}^{\mathrm{T}}. \tag{2f}$$

In our case where we have three EGMs, after further simplifications, $\mathbf{B}$ becomes:

$$\mathbf{B} = \left[\frac{p_{nm}^{2}\left(t_{nm}^{1} - t_{nm}^{2}\right) + p_{nm}^{3}\left(t_{nm}^{1} - t_{nm}^{3}\right)}{\left(p_{nm}^{1} + p_{nm}^{2} + p_{nm}^{3}\right)^2} \quad \frac{p_{nm}^{2}\left(t_{nm}^{2} - t_{nm}^{1}\right) + p_{nm}^{3}\left(t_{nm}^{2} - t_{nm}^{3}\right)}{\left(p_{nm}^{1} + p_{nm}^{2} + p_{nm}^{3}\right)^2} \quad \frac{p_{nm}^{2}\left(t_{nm}^{3} - t_{nm}^{1}\right) + p_{nm}^{3}\left(t_{nm}^{3} - t_{nm}^{2}\right)}{\left(p_{nm}^{1} + p_{nm}^{2} + p_{nm}^{3}\right)^2} \right]. \tag{2g}$$

Since the condition model was linearised it should be applied iteratively. Therefore the minimum norm estimation of $\Delta\mathbf{p}$ is:

$$\widetilde{\Delta\mathbf{p}}^{k+1} = \mathbf{B}_k^{\mathrm{T}}\left(\mathbf{B}\mathbf{B}_k^{\mathrm{T}}\right)^{-1} \Delta l^{k} \tag{2h}$$