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Effects of source and cavity depths on wave fields in homogeneous half spaces

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ABSTRACT

In homogeneous half spaces excited by small buried, spherically symmetric P-wave sources, Rayleigh waves (R-waves) could be generated. The effects of the source depth on the induced wave pattern and propagation behavior of R-waves are analyzed using the thin layer method. When a cavity is present in a homogeneous half space, R-waves could be formed in the scattered wave field. It is found that the energy of R-waves in the incident (direct) surface wave-field is related to the ratio of the source depth to the wavelength of R-waves; R-waves have relatively strong energy when the ratio is less than 1; the buried source induced R-waves approximately travel at the velocity of the planar R-waves in the range of the offset beyond about one wavelength; the energy of R-waves in the back-scattered surface wave-field depends on the ratio of the depth of cavity to the wavelength of R-waves; and for the case of the cavity presented at depths less than one wavelength, R-waves can be clearly observed in the back-scattered wave field. The results are helpful for selecting the source depth and the frequency component in seismic surveys and interpreting both the incident wave and the scattered wave patterns.

1. Introduction

When heterogeneities are present in a medium, the body (P- and S-) and R-waves could be scattered at the boundaries of the heterogeneities. These heterogeneities can be potentially characterized from the scattered waves (António and Tadeu, 2001; Boström and Kristensson, 1983; Furlong et al., 1994; Grandjean and Leparoux, 2004; Höllinger and Ziegler, 1979; Lüth et al., 2008; Riyanti, 2005; Scales, 1994; Tadeu et al., 2002, 2006; Virieux, 1986; Vallamsundar, 2007; Vogelaar, 2001; Xia et al., 2007). The seismic waves could be generated by surface or buried sources. These could be active sources, such as impact loadings caused by a sledge-hammer/weight drop or an explosion, or passive sources associated with ambient noise and micro-tremors (Nasseri-Moghaddam et al., 2007; Morikawa et al., 2004; Park et al., 2005; Tokimatsu et al., 1992). The appropriate source type depends on the depth of the suspected heterogeneities and the dominant components of the waves it could induce. The majority of energy of the wave field induced by surface sources is carried by R-waves (Graff, 1975). Energy of R-waves is concentrated within a depth of about one wavelength, i.e. penetration depth of R-waves is about one wavelength. When a cavity is located within the penetration depth of R-waves and far from the source in the horizontal direction, a large part of the scattering is caused by incident R-waves. By analyzing changes in the wave patterns both in time and frequency (wavelength) domains, a shallow cavity can be approximately located (Chai et al., 2012b). However, for cavities located at depths beyond 50 m, sources with sufficiently low dominant frequency (at several Hz or smaller) are required to produce R-waves capable of reaching that depth. The waves propagating at such low frequencies are difficult to be generated using active sources. Passive sources are often used in such cases (Morikawa et al., 2004; Park et al., 2005; Tokimatsu et al., 1992). Since the incident direction of passive waves is unknown or the passive waves are omnidirectional and the time dependent statistical nature of passive waves (Hebeler and Rix, 2001), both experimental and analysis methods must be developed for the characterization of subterranean heterogeneities using this kind of waves. At present, it is inconvenient to detect cavities at deep depths (>50 m) using R-waves.

Compared to R-waves, the body waves, as its name implies, can propagate in the body of a solid medium in any direction. The body waves can reach deep subterranean features. However, the energy of body waves induced by the surface sources is relatively small compared to R-waves. The scattered body waves in the surface wave-field could be weak due to the divergence (geometrical attenuation) of the body waves to and from the heterogeneities and obscured by R-waves. The wave field induced by buried sources is entirely controlled by the body waves before these waves reach the free surface. Therefore, the energy of incident body waves can be enhanced using buried sources. When the incident body waves reach the free surface, R-waves could be formed by interference of the incident waves and their reflections at the surface under some conditions

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(Ewing et al., 1957). R-waves generated by underground explosions have been observed (Day et al., 1983). The proportion of R-waves in the surface wave field is influenced by the ratio of the source depth to the wavelength of the R-waves. Although some integral displacement expressions are available for buried sources (Anderson, 2011; Madariaga, 2007; Pinney, 1954), it is not straightforward to analyze effects of the source depth on the incident wave field from these implicit and complex expressions.

According to Huygens' Principle, each point on a wavefront can be considered as a secondary source. R-waves maybe are formed in the scattered wave field. The presence of R-waves may severely mask reflection events (Bohlen et al., 2003; Campman, 2005; Campman et al., 2005). Thus, by investigating the influence of the source depth and the cavity depth on the surface wave fields, some heterogeneity detection guidelines can be developed for (i) selecting the appropriate depth and the dominant frequency range of the source and (ii) analyzing the wave patterns of both the incident and the scattered wave fields.

When the sizes of the buried spherical P-wave sources are small relative to the buried depth, it is reasonable to treat the buried sources as spherical point sources. In this work, the discrete explicit displacement expression of the incident R-waves is first established for buried spherical point sources based on the thin layer method (Kausel, 1999, 2001; Kausel and Peek, 1982; Kausel and Roësset, 1981). The effects of the dimensionless source depth (i.e. the ratio of source depth to the wavelength of R-waves) on the R-waves in the incident wave field are then analyzed. Next, the propagation behavior of induced R-waves is studied and compared with that of the planar R-waves. Finally, the relationship between R-waves in the back-scattered wave field and the dimensionless scattering depth is presented and analyzed.

2. Scattered wave field

In a layered half space containing heterogeneities and a point source acting at the position $\mathbf{x}_s$ in direction k ($k = 1, 2, 3$, the indices 1, 2 and 3 refer to the variables or derivatives with respect to x, y and z coordinates, respectively), the i-th ($i = 1,\ 2,\ 3$) component of the full displacement $u_i(\mathbf{x,x}_s)$ at the position $\mathbf{x}$ ($\mathbf{x} \in \Omega \cup S$ shown in Fig. 1) can be divided into two parts as (Campman, 2005; Riyanti, 2005)

$$u_i(\mathbf{x}, \mathbf{x}_s) = u_i^{inc}(\mathbf{x}, \mathbf{x}_s) + u_i^{sc}(\mathbf{x}, \mathbf{x}_s), \tag{1}$$

where

$$u_i^{inc}(\mathbf{x}, \mathbf{x}_s) = W(\omega) u_{ik}^G(\mathbf{x}, \mathbf{x}_s), \tag{2}$$

$$\begin{aligned} u_i^{sc}(\mathbf{x}, \mathbf{x}_s) = {} & \omega^2 \int_{\mathbf{x}' \in \Omega} \left[\rho^A\left(\mathbf{x}'\right) - \rho^B\left(\mathbf{x}'\right)\right] u_{ik}^G\left(\mathbf{x}, \mathbf{x}'\right) u_k\left(\mathbf{x}', \mathbf{x}_s\right) dV \\ & + \int_{\mathbf{x}' \in \Omega} \left[s_{kjpq}^A\left(\mathbf{x}'\right) - s_{pqkj}^B\left(\mathbf{x}'\right)\right] \tau_{pqi}^G\left(\mathbf{x}', \mathbf{x}\right) \tau_{kj}\left(\mathbf{x}', \mathbf{x}_s\right) dV, \end{aligned} \tag{3}$$

in which $u_i^{inc}(\mathbf{x,x}_s)$ denotes the displacement of the incident wave field generated by the source at the position $\mathbf{x}_s$ in the layered media without any heterogeneities and $u_i^{sc}(\mathbf{x,x}_s)$ represents the displacement of the scattered wave field in the presence of the heterogeneity D; ω is angular frequency; $W(\omega)$ denotes the source function in the frequency domain; $u_{ik}^G(\mathbf{x,x}_s)$ is the i-th component of Green's displacement due to a point pulse source acting in the k-th direction at the position $\mathbf{x}_s$; $u_{ik}^G\left(\mathbf{x}, \mathbf{x}'\right)$ and $\tau_{pqk}^G\left(\mathbf{x}, \mathbf{x}'\right)$ are the i-th component of Green's displacement and the stress tensor at the position $\mathbf{x}$ due to a point pulse source directed in the k-th direction at the position $\mathbf{x}'$, respectively; $\tau_{kj}\left(\mathbf{x}', \mathbf{x}_s\right)$ is the component of the stress tensor of the state A at position vector $\mathbf{x}'$; $\rho^A(\mathbf{x})$ is the density of the state A; $s_{ijpq}^A(\mathbf{x})$ and $s_{ijpq}^B(\mathbf{x})$ are the fourth rank compliance tensors of states A and B, respectively. The tensors are related to the Lamé's constants. Since changes in the Lamé's constants affect the pattern of the scattered waves in the same way as those caused by changes in the density (Campman, 2005), only density contrast is considered to analyze the scattered R-wave field in the following analysis. By ignoring the influence of any contrast due to Lamé's constants, the displacements of the scattered wave field can be simplified as

$$u_i^{sc}(\mathbf{x}, \mathbf{x}_s) \approx \omega^2 \int_{\mathbf{x}' \in D} \left[\rho^A\left(\mathbf{x}'\right) - \rho^G\left(\mathbf{x}'\right)\right] u_{ik}^G\left(\mathbf{x}, \mathbf{x}'\right) u_k\left(\mathbf{x}', \mathbf{x}_s\right) dV. \tag{4}$$

The Green's functions in homogeneous or layered half spaces in Eqs. (2) to (4) can be expressed in many different ways (Anderson, 2011; Campman, 2005; Campman et al., 2005; Kausel, 1999; Riyanti, 2005; Tadeu et al., 2002, 2006). However, the contributions of different types of waves (the body and Rayleigh waves) to the displacements are not identified and separated in most of these expressions of the Green's functions. As such, it may be quite inconvenient to analyze effects of the source and the cavity depths on R-waves using these expressions. The discrete Green's functions obtained from the thin layer method (Kausel, 1999) are presented in terms of modal displacements which correspond to the body waves, the actual R-waves and the Love waves, respectively. Thus, from the discrete Green's expressions, the relationship between modal displacement distribution in the depth direction and the source and the cavity depths can be established. In this way, the effects of the source and the cavity depths on the incident and the scattered wave fields can be analyzed.

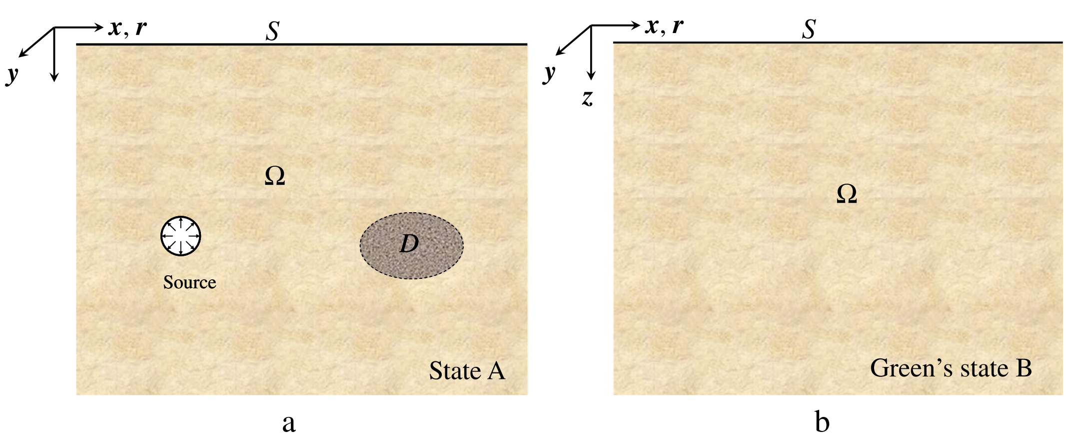

Fig. 1. Actual state A and Green's state: (a) with a buried source and a heterogeneity; and (b) without any heterogeneities.