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Interpretation of gravity data by the continuous wavelet transform: The case of the Chad lineament (North-Central Africa)

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ABSTRACT

A slightly bended gravity high along the Chad lineament in Central North Africa is analyzed and interpreted by the continuous wavelet transform (CWT) method. We use scale normalization on the continuous wavelet transform, allowing analysis of the gravity field in order to determine the sources at different depths. By focusing on homogenous standard sources, such as sphere or cube, horizontal cylinder or prism, sheet and infinite step, we derive the relationships between the source depth and pseudo-wavenumber. Then the source depth can be recovered from tracing the maximal values of the modulus of the complex wavelet coefficients in the CWT-based scalograms that are function of the pseudo-wavenumber. The studied area includes a central gravity high up to 75 km wide, and a secondary high that occurs at the southern part of the anomaly. The interpretation of the depth slices and vertical sections of the modulus maxima of the complex wavelet coefficients allows recognition of a relatively dense terrane located at middle crustal levels (10–25 km depth). A reasonable geological model derived from the 2.5D gravity forward modelling indicates the presence of high density bodies, probably linked to a buried suture, which were thrusted up into the mid-crust during the Neo-Proterozoic terrane collisions between the Saharan metacraton and the Arabian-Nubian shield. We conclude that the Chad line delineates a first order geological boundary, missing on the geologic maps.

1. Introduction

The Chad lineament, a striking arched gravity anomaly in eastern Chad, had been noticed in the early 70s: It has been first mentioned by Louis (1970), who interprets the anomaly to be caused by subcrustal material that was transferred to upper crustal levels. Louis (1970) suggests that the age of the lineament is older than Cretaceous, and it is probably related to a consolidated fracture with the presence of heavy elements of Caledonian or Precambrian age, which is contemporaneous to the Pan African Trans-Saharan suture. In Burke and Whiteman (1973), the poli triple-rift junction (centered at 9°N, 14°E), including the Chad anomaly, is suggested to mark a rift (termed Ati rift) forming on arm of the Poli structure, whose age approximates at 130–80 Ma. The positive gravity anomaly is interpreted as a long series of basic intrusions. Following the hypothesis of Burke and Whiteman (1973), Freeth (1984) proposed that an exceptionally and improbable great dimension of the crustal dyke is needed to explain the gravity observations (over 1000 km length, 35 km width, near to 30 km thickness), which is covered by a band of basic volcanic and/or volcanoclastic sediments. In Fairhead and Green (1989), the Chad gravity anomaly is generally described within the context of lithospheric extension and basin formation (Mckenzie, 1978), where stretching of the lithosphere results in passive upwelling of hotter, less dense asthenosphere and a concomitant necking of the low density crust. A recent review on metacratons refers to the gravity anomaly described in Braitenberg et al. (2011b) and proposes it to coincide with the eastern border of a hypothetical Chad craton, a remnant of the pre-Neoproterozoic Saharan craton (Liégeois et al., 2012).

The new global gravity models, derived from integrating terrestrial with satellite data (Pavlis et al., 2012), or derived from the satellite GOCE (Floberghagen et al., 2011), that have unprecedented precision and spatial resolution offer a new opportunity to interpret the Chad lineament. In Braitenberg et al. (2011b), the characteristic of the linear gravity anomaly from EGM2008 (Pavlis et al., 2012) and GOCE model (Migliaccio et al., 2010) was compared with that of the Central African rift and the Pan-African suture. Their conclusion is that the lineament differs from the nearby rifts in its signature of gravity and topography, and thus is not coeval and most likely is an old suture of the Saharan metacraton. However, the interpretation of potential field data is not a straight-forward process because of the many models capable of explaining the observed field. For the Chad gravity high, its origin is still a mystery due to limited geophysical constraints. If located in the upper crust, it is likely a 5 km thick body, 100 km width, 1200 km long. If in the lower crust, the body may be up to 180 km wide, 15 km thick (Braitenberg et al., 2011b). These are only two of many possible models that can explain the anomaly.

A sophisticated potential field analysis technique has been developed to help in reducing the well-known non-uniqueness problem

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by adding a priori constraints. The continuous wavelet transform (CWT) that appears in potential field interpretation in the 90s is one of the techniques that can simplify the fast analysis of large amounts of data (Sailhac et al., 2009).

Ridsdill-Smith and Dentith (1999) have developed the application of the wavelet transform for processing aeromagnetic data. With a specific family of wavelets, Hornby et al.(1999) analyze potential field data to locate singular features of the source distribution. Using the same family of wavelets, Moreau et al. (1997, 1999), Sailhac et al. (2000), and Martelet et al. (2001) develop another interpretation technique based on continuous wavelet theory. Their technique can estimate the source position and type, assuming the sources are homogeneous. The depth and structural index are estimated by successively testing the least-squares misfit between a straight line and the wavelet coefficients plotted against the scale in log-log space. This technique, developed for homogeneous sources, has been generalized to multiple sources and extended sources of finite size and dipping angles (finite step, thin and thick dikes, prisms) by Sailhac et al. (2000), Martelet et al. (2001), and Sailhac and Gibert (2003). As an extension of the technique of Hsu et al. (1998) and Sailhac et al. (2000), Vallée et al. (2004) proposed an alternative technique that estimates the depth and the structural index of multiple (nonhomogeneous) sources from the ratio of wavelets of successive orders. Yang et al. (2010) devised a scale normalization scheme on the continuous wavelet transform to facilitate the source estimation for superimposed magnetic anomalies. For a better delineation of source depths, a linear relationship between the source depth and the wavelet pseudo-wavenumber is developed from a synthetic modeling.

In this paper we study the gravity anomaly derived from EGM2008 potential field model (Pavlis et al., 2012) in the area of the Chad linement, north-central Africa, based on the scale-normalized continuous wavelet transform (Li et al., 2011; Yang et al., 2010).

2. Method

2.1. The scale-normalized continuous wavelet transform (CWT)

We recall the theory developed by Mallat (1999), applied in a 2D physical space. The continuous wavelet transform (CWT) of signal, $s \in L^2(R)$, is defined as the integral transform $\tilde{S}_W$ (Mallat, 1999)

$$\tilde{S}_W(a,b) = \int_{-\infty}^{\infty} s(x) \frac{1}{\sqrt{a}} \overline{\psi}\left(\frac{x-b}{a}\right) dx, \tag{1}$$

where $\overline{\psi}$ is the complex conjugate of a fixed function $\psi \in L^2(R)$, called the mother wavelet or analyzing wavelet; $a \in R^+$ and $b \in R$ are the scale and the translation parameter, respectively, with R^+ being the set of positive real numbers. $L^2(R)$ denotes the Hilbert space of square integrable functions. $\tilde{S}_W$ (a, b) is the scalogram (wavelet coefficients).

For the purpose of enhancing frequency discrimination ability for superimposed magnetic anomalies, Yang et al. (2010) designed a scale normalization scheme, applying a power function of scale (a^{-n}) on the 1-D continuous wavelet transform (Eq. (1)). Then the scale-normalized 1-D continuous wavelet transform will be expressed as

$$\tilde{S}'_W(a,b) = a^{-n} \int_{-\infty}^{\infty} s(x) \frac{1}{\sqrt{a}} \overline{\psi}\left(\frac{x-b}{a}\right) dt \tag{2}$$

where n is a positive constant. When $n=0$, there is no scale normalization.

Therefore, the gravity anomalies in different profiles can be transformed in the continuous wavelet domain to obtain their scalograms by Eq. (2), indicating the modulus of complex wavelet coefficients in the plane of scale against the distance. We note that a scale represents a range of depth or a wavenumber band, and the scalogram cannot provide a direct indication of depth. To interpret the scalogram, we stretch the scale to an equivalent wavenumber, depending on the scale-wavenumber mapping of the wavelet.

The center wavenumber k_c of a signal s is defined as:

$$k_c = \frac{\int_0^{\infty} k |\tilde{s}(k)|^2 dk}{\int_0^{\infty} |\tilde{s}(k)|^2 dk}, \tag{3}$$

where $\tilde{s}(k)$ is the Fourier transform of a signal. The pseudo-wavenumber k_a corresponding to the scale a and the sampling period Δ is defined as:

$$k_a = \frac{k_{c,w}}{a \cdot \Delta} \tag{4}$$

Once the mother wavelet is chosen, its center wavenumber $k_{c,w}$ can be determined from Eq. (3). Then the scales in the scalogram can be transformed into pseudo-wavenumbers k_a by Eq. (4).

2.2. Discussion on the relationship between depth and pseudo-wavenumber

For source depth estimation, we need to establish the bridge to convert the pseudo-wavenumber to the source depth. Note that for a magnetic sphere model, the pseudo-wavenumber $k_{a,n}{}^{(\max)}$ corresponding to the modulus maximum of complex coefficients after being transformed by the scale-normalized CWT, always has a linear relationship with the source depth $k_{a,n}^{(\max)} = \frac{0.8(n+1)}{3} h$ (Yang et al., 2010). For gravity anomalies, a similar linear relationship also exists between the pseudo-wavenumber and the source depth. Before elaborating on a synthetic example, we give a brief discussion on the choice of wavelet function in the CWT.

Complex wavelets can easily be constructed from real wavelets through the Hilbert transform. There are four kinds of commonly used complex wavelets: complex Morlet wavelets, complex Gaussian wavelets, complex frequency B-spline wavelets and complex Shannon wavelets. Among them, the complex Morlet wavelets, whose Fourier transform is a Gaussian function, are a fairly ideal band-pass filter. Therefore, it is adopted in our work for the spectral analysis.

The complex Morlet mother wavelet is defined as:

$$\psi(x) = \sqrt{\pi k_b} e\left(-x^2/k_b + 2i\pi k_c x\right), \tag{5}$$

where k_b is the bandwidth parameter and k_c is the wavelet centre wavenumber. By dilating and translating this wavelet$\psi(x)$, we produce a family of wavelets:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi\left(\frac{x-b}{a}\right). \tag{6}$$

Table 1
The model geometry parameters and the calculated pseudo-wavenumber $k_{a,2}{}^{(\max)}$ corresponding to modulus maxima of complex wavelet coefficients.

Center depth (km)	Radius (km)	Total mass (10^8 kg)	Pseudo-wavenumber $k_{a,2}^{(\max)}$ (km^{-1})
0.5	0.05	2.618	1.04
1	0.1	20.944	0.51
2	0.2	167.552	0.261198
3	0.3	565.4878	0.18
4	0.4	1340.413	0.133859
5	0.5	2617.994	0.11
6	0.6	4523.893	0.09
7	0.7	7183.775	0.076065
8	0.8	10723.303	0.065
9	0.9	15268.140	0.06
10	1	20943.951	0.055
20	2	167551.608	0.026
30	3	565486.678	0.018
40	4	1340412.866	0.014
50	5	2617993.878	0.012