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A generalized optimal 9-point scheme for frequency-domain scalar wave equation

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ABSTRACT

The rotated optimal 9-point scheme for frequency-domain scalar wave equation is widely used in frequency-domain full waveform inversion. This scheme requires equal directional sampling intervals, which limits its applicability. Recently, an average-derivative method was proposed to overcome this restriction. However, the average-derivative method is an algebraic approach, and therefore it does not inherit the geometrical property (coordinate transformations) of the rotated optimal 9-point scheme. In this paper, a geometrical approach is developed, and a generalized optimal 9-point scheme is constructed. This new scheme is based on a directional-derivative method, and includes the rotated optimal 9-point scheme as a special case. Like the average-derivative method, the number of grid points per wavelength is reduced from approximately 13 to approximately 4 by this new 9-point optimal scheme for both equal and unequal directional sampling intervals in comparison with the classical 5-point scheme. Unlike the average-derivative method, this generalized optimal 9-point scheme shares the geometrical property of the rotated optimal 9-point scheme.

1. Introduction

Recently, full waveform inversion (FWI) has been attracting a lot of attention in community of exploration geophysics. Generally speaking, FWI can be described as a full-wavefield-modeling-based data-fitting process to extract structural information of subsurface from seismograms (Virieux and Operto, 2009). FWI can be classified into two categories: time-domain FWI (Boonyasiriwat et al., 2009; Gauthier et al., 1986; Tarantola, 1984) and frequency-domain FWI (Pratt, 1999; Pratt and Worthington, 1990; Pratt et al., 1998).

Forward modeling is an important foundation of FWI. In the context of FWI, Pratt and Worthington (1990) developed the classical 5-point scheme for 2D frequency-domain scalar wave equation which imposes no restriction on directional sampling intervals. However, this scheme suffers from severe dispersion errors when large sampling intervals (4 points per smallest wavelength) are employed. To reduce the dispersion errors, very small sampling intervals (13 points per smallest wavelength) are required, which results in a significant increase of both storage requirements and CPU time.

Based on a rotated coordinate system, Jo et al. (1996) developed a 9-point operator to approximate the Laplacian and the mass acceleration terms. The coefficients of the 9-point operator are determined by obtaining the best normalized phase curves through an optimization process. Compared to the classical 5-point scheme developed by Pratt and Worthington (1990), this optimal 9-point scheme reduces the number of grid points per wavelength to less than 4, and leads to significant reductions of computer memory and CPU time. However, this optimal 9-point scheme loses the flexibility of the classical 5-point scheme because it requires equal directional sampling intervals (Jo et al., 1996).

To overcome the disadvantage of the rotated optimal 9-point scheme, Chen (2012) developed an average-derivative method. Unlike the method used by Jo et al. (1996), the average-derivative method does not need to use rotated coordinate system and only involves algebraic operations. In this paper, I will develop another approach to overcome the disadvantage of the rotated optimal 9-point scheme. This approach is based on the directional-derivative method proposed by Saenger et al. (2000). The directional-derivative method is closely related to the rotated-coordinate-system method, but has more flexibility.

In the next section, I will present the generalized optimal 9-point scheme based on the directional-derivative method and staggered-grid technique. This is followed by optimization of coefficients and a numerical dispersion analysis. Finally, I perform two numerical experiments on a homogenous model and the Marmousi model to test the generalized optimal 9-point scheme.

2. Generalized optimal 9-point scheme

Consider the two-dimensional scalar wave equation in frequency domain

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial z^2}+\frac{\omega^2}{v^2}P=0, \tag{1}$$

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where P is the pressure wavefield, ω is the angular frequency, and $v(x,z)$ is the velocity.

The classical 5-point scheme for Eq. (1) is

$$\frac{P_{m+1,\ n}-2P_{m,\ n}+P_{m-1,\ n}}{\Delta x^2}+\frac{P_{m,\ n+1}-2P_{m,\ n}+P_{m,\ n-1}}{\Delta z^2}+\frac{\omega^2}{v_{m,\ n}^2}P_{m,\ n}=0, \tag{2}$$

where $P_{m,\ n}\approx P(m\Delta x,n\Delta z)$, and Δx and Δz are directional sampling intervals in the x-direction and z-direction, respectively.

As can be seen later (Section 3), within the phase velocity error of $\pm 1\%$, the classical 5-point scheme (2) requires approximately 13 grid points per shortest wavelength. In order to reduce numerical dispersion of the scheme (2), very fine grids are required. This leads to a huge amount of computer storage and CPU time. Therefore, reducing the number of grid points required per shortest wavelength is needed.

To this aim, a 9-point scheme for Eq. (1) was introduced by Jo et al. (1996):

$$\begin{aligned}&a\frac{P_{m+1,\ n}+P_{m-1,\ n}-4P_{m,\ n}+P_{m,\ n+1}+P_{m,\ n-1}}{\Delta^2}\\&+(1-a)\frac{P_{m+1,\ n+1}+P_{m-1,\ n+1}-4P_{m,\ n}+P_{m+1,\ n-1}+P_{m-1,\ n-1}}{2\Delta^2}\\&+\frac{\omega^2}{v^2}\Big(cP_{m,\ n}+d\big(P_{m+1,\ n}+P_{m-1,\ n}+P_{m,\ n+1}+P_{m,\ n-1}\big)\\&+e\big(P_{m+1,\ n+1}+P_{m-1,\ n+1}+P_{m+1,\ n-1}+P_{m-1,\ n-1}\big)\Big)=0,\end{aligned} \tag{3}$$

where $\Delta x=\Delta z=\Delta$. The constants a, c and d are weighting coefficients, and $e=\frac{1-c-4d}{4}$. For details, See Fig. 1a.

The rotated 9-point optimal scheme (3) with coefficients ($a=0.5461$, $c=0.6248$, and $d=0.0938$) reduces the number of grid points per shortest wavelength to less than 4, and results in remarkable reductions of computer storage and CPU time. However, this scheme has a requirement of $\Delta x=\Delta z$, which limits its application.

Now I try to develop a generalization of scheme (3) which is also valid for $\Delta x\neq\Delta z$. When $\Delta x\neq\Delta z$, the idea of rotated coordinate system can be developed into the directional-derivative method (Saenger et al., 2000). For details, see Fig. 1b. When $\Delta x\neq\Delta z$, the two directions l_1 and l_2 are not orthogonal to each other. One can compute directional-derivatives as follows:

$$\frac{\partial P}{\partial l_1}=\frac{\Delta x}{\Delta r}\frac{\partial P}{\partial x}-\frac{\Delta z}{\Delta r}\frac{\partial P}{\partial z}, \tag{4}$$

$$\frac{\partial P}{\partial l_2}=\frac{\Delta x}{\Delta r}\frac{\partial P}{\partial x}+\frac{\Delta z}{\Delta r}\frac{\partial P}{\partial z}, \tag{5}$$

where $\Delta r=\sqrt{\Delta x^2+\Delta z^2}$. From Eqs. (4) and (5), one can obtain

$$\frac{\partial^2 P}{\partial l_1^2}+\frac{\partial^2 P}{\partial l_2^2}=2\left(\frac{\Delta x}{\Delta r}\right)^2\frac{\partial^2 P}{\partial x^2}+2\left(\frac{\Delta z}{\Delta r}\right)^2\frac{\partial^2 P}{\partial z^2}, \tag{6}$$

$$\frac{\partial^2 P}{\partial l_1\partial l_2}=\left(\frac{\Delta x}{\Delta r}\right)^2\frac{\partial^2 P}{\partial x^2}-\left(\frac{\Delta z}{\Delta r}\right)^2\frac{\partial^2 P}{\partial z^2}. \tag{7}$$

From Eqs. (6) and (7), one can further obtain

$$\begin{aligned}\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial z^2}=&\left(\frac{\Delta r^2}{4\Delta x^2}+\frac{\Delta r^2}{4\Delta z^2}\right)\left(\frac{\partial^2 P}{\partial l_1^2}+\frac{\partial^2 P}{\partial l_2^2}\right)\\&+\left(\frac{\Delta r^2}{2\Delta x^2}-\frac{\Delta r^2}{2\Delta z^2}\right)\frac{\partial^2 P}{\partial l_1\partial l_2}.\end{aligned} \tag{8}$$

From Eq. (8), an approximation to the Laplacian can be obtained:

$$\begin{aligned}\frac{\partial^2 P}{\partial x^2}(m,n)+\frac{\partial^2 P}{\partial z^2}(m,n)=&\frac{P_{m+1,\ n+1}+P_{m-1,\ n+1}-4P_{m,\ n}+P_{m+1,\ n-1}+P_{m-1,\ n-1}}{2\tilde{\Delta}^2}\\&+\frac{\Delta z^2-\Delta x^2}{8\Delta x^2\Delta z^2}\left(P_{m+2,\ n}-P_{m,\ n+2}-P_{m,\ n-2}+P_{m-2,\ n}\right)\\&+\mathcal{O}\left((\Delta x,\Delta z)^2\right),\end{aligned} \tag{9}$$

where $\tilde{\Delta}=\frac{1}{\sqrt{\frac{1}{2}\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta z^2}\right)}}$. $\tilde{\Delta}$ can be called the root-harmonic-mean-square interval of Δx and Δz (Chen, 2011).

Using Eq. (9), one can obtain the following scheme

$$\begin{aligned}&a\left[\frac{P_{m+1,\ n}-2P_{m,\ n}+P_{m-1,\ n}}{\Delta x^2}+\frac{P_{m,\ n+1}-2P_{m,\ n}+P_{m,\ n-1}}{\Delta z^2}\right]\\&+(1-a)\left[\frac{P_{m+1,\ n+1}+P_{m-1,\ n+1}-4P_{m,\ n}+P_{m+1,\ n-1}+P_{m-1,\ n-1}}{2\tilde{\Delta}^2}\right.\\&\left.+\frac{\Delta z^2-\Delta x^2}{8\Delta x^2\Delta z^2}\left(P_{m+2,\ n}-P_{m,\ n+2}-P_{m,\ n-2}+P_{m-2,\ n}\right)\right]\\&+\frac{\omega^2}{v^2}\Big(cP_{m,\ n}+d\big(P_{m+1,\ n}+P_{m-1,\ n}+P_{m,\ n+1}+P_{m,\ n-1}\big)\\&+e\big(P_{m+1,\ n+1}+P_{m-1,\ n+1}+P_{m+1,\ n-1}+P_{m-1,\ n-1}+A\big)\Big)=0.\end{aligned} \tag{10}$$

where

$$A=\begin{cases}0, & \Delta x=\Delta z,\\ P_{m+2,\ n}-P_{m,\ n+2}-P_{m,\ n-2}+P_{m-2,\ n}, & \Delta x\neq\Delta z.\end{cases}$$

Scheme (10) is a 13-point scheme. Compared to the rotated optimal 9-point scheme, it includes four additional grid points. This is caused by the second-order mixed partial derivative in Eq. (8). When $\Delta x=\Delta z$, the term involving the mixed partial derivative becomes zero, and the 13-point scheme becomes the rotated optimal 9-point scheme.

On the other hand, without the condition of $\Delta x=\Delta z$, the scheme (10) can be simplified too. Using a staggered-grid technique (Štekl and Pratt, 1998), the second term on the right-hand side of Eq. (8) can be discretized as follows:

$$\left(\frac{\Delta r^2}{2\Delta x^2}-\frac{\Delta r^2}{2\Delta z^2}\right)\frac{\partial^2 P}{\partial l_1\partial l_2}\approx\frac{\Delta z^2-\Delta x^2}{2\Delta x^2\Delta z^2}\left(P_{m+1,\ n}-P_{m,\ n+1}-P_{m,\ n-1}+P_{m-1,\ n}\right). \tag{11}$$

Using the approximation (11), the 13-point scheme (10) can be simplified into a 9-point scheme:

$$\begin{aligned}&a\left[\frac{P_{m+1,\ n}-2P_{m,\ n}+P_{m-1,\ n}}{\Delta x^2}+\frac{P_{m,\ n+1}-2P_{m,\ n}+P_{m,\ n-1}}{\Delta z^2}\right]\\&+(1-a)\left[\frac{P_{m+1,\ n+1}+P_{m-1,\ n+1}-4P_{m,\ n}+P_{m+1,\ n-1}+P_{m-1,\ n-1}}{2\tilde{\Delta}^2}\right.\\&\left.+\frac{\Delta z^2-\Delta x^2}{2\Delta x^2\Delta z^2}\left(P_{m+1,\ n}-P_{m,\ n+1}-P_{m,\ n-1}+P_{m-1,\ n}\right)\right]\\&+\frac{\omega^2}{v^2}\Big(cP_{m,\ n}+d\big(P_{m+1,\ n}+P_{m-1,\ n}+P_{m,\ n+1}+P_{m,\ n-1}\big)\\&+e\big(P_{m+1,\ n+1}+P_{m-1,\ n+1}+P_{m+1,\ n-1}+P_{m-1,\ n-1}\big)\Big)=0.\end{aligned} \tag{12}$$

The scheme (12) is a generalized optimal 9-point scheme because it includes the rotated optimal 9-point scheme (3) as a special case