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Inversion of geo-magnetic full-tensor gradiometer data

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ABSTRACT

The fast and sensitive SQUID (Superconducting Quantum Interference Device) system, which was developed at IPHT Jena, allows the geo-magnetic prospection of large land areas. The system's simultaneous high-resolution recording of all components of the Earth's magnetic field gradient tensor represents a high-quality data base for precise inversion calculations. Thus, we developed a software tool for the fast and direct inversion of full-tensor data from especially dipole-like sources. Our motivation is to localize buried magnetic objects and inhomogeneities in the underground only by measuring the gradient components at the surface. The application of the algorithm will be shown by two examples, first on a synthetic data set and second on a real data set measured at the IPHT test site with well-defined buried targets.

1. Introduction

The aim of new geophysical prospection methods is to cover large areas of interest to illustrate the hidden sub-surface structure. But, conventional methods like drill hole and excavation analytics are longsome, costly and destructive. Therefore, many fast and non-destructive methods using optical (Ricchetti, 2004), acoustical and electromagnetic techniques (Linford, 2006) gather more and more interest because they are non-invasive as well as eco-friendly. In comparison to invasive methods which provide only the supporting points to two-dimensional model based descriptions of the underground, magnetic and electromagnetic analytics can produce complete two-dimensional maps of the investigated areas.

In this work, we apply a system which uses a very fast and ultra-sensitive magnetic prospecting technology based on low temperature superconducting SQUIDs. This system enables a new data quality of reconstructing underground structures out of its mapping results. The magnetic properties of buried objects and their contrast to the surrounding host material cause anomalies of the Earth's magnetic field which produces gradients within the homogeneous underground field. These anomalies are mapped with the introduced passive measurement system. As opposed to other established methods of magnetic prospecting – like fluxgate measurements for instance (Munschy et al., 2007) – a system of planar-type SQUID gradiometers, called FTMG system herein, measures the full gradient tensor information of the local variations of Earth's magnetic field described in detail in Stolz et al. (2006). The system was deployed successfully already in air- and seaborne operation. Here, we focus on the ground-based system which enables the mapping of extended regions for instance in application fields like archeology (Linzen et al., 2009), building-ground analytics, geology and detection of UXO (unexploded ordnance) (Meyer et al., 2009). The system is equipped additionally with a differential GPS (Global Positioning System) with base station for high precision positioning and an inertial unit which provides attitude angle information for motion noise compensation of the direction sensitive SQUID sensor signals.

The described system delivers high resolution magnetic field gradient tensor data which are well suited for inversion approaches of dipole-like sources. In this work we will show our implementation of an algorithm based on the US patent from 1998 by Wynn (1998) and an article from 1972 written by Frahm (1972) which we call Wynn Frahm algorithm and short WFA herein.

Based on the mentioned proposals, the knowledge of the magnetic field gradient tensor information combined with position and orientation of the sensor, the dipole-like appearance of the source and the local, nearly homogeneous magnetic ambient field are needed to ensure the applicability of the WFA. In comparison to other proposals (Beiki and Pedersen, 2011; Furness, 1994), the WFA approach has no complex descriptions of two or three dimensional buried structures, which would be adapted numerically to the measured data. The description of the dipole-like sources in the presented work is based on well-known

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fundamentals resulting from Maxwell's equation for magneto-static conditions (Jackson, 1985). Here, the magnetic field is defined by

$$\vec{B}=\frac{\mu_0}{4\pi}\left(\frac{3\left(\vec{m}\cdot\vec{r}\right)\,\vec{r}}{|\vec{r}|^5}-\frac{\vec{m}}{|\vec{r}|^3}\right), \quad (1)$$

where $\vec{m}$ is the magnetic moment vector of the source and $\vec{r}$ the bearing vector between sensor and source position. Subsequently, the needed gradient tensor components result by constructing the Jacobian matrix out of this term teBronstein1999. It is formally presented in Eq. (2).

$$Jacobian\left(\vec{B}\right)=\begin{pmatrix}\frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z}\\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z}\\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z}\end{pmatrix}=\begin{pmatrix}G_{11} & G_{12} & G_{13}\\ G_{21} & G_{22} & G_{23}\\ G_{31} & G_{32} & G_{33}\end{pmatrix}=\left(\nabla\circ\vec{B}\right) \quad (2)$$

Supplemental to known advantages of magnetic gradient and full tensor measurements (Schmidt and Clark, 2000), the direct and independent measurement of all the five tensor components in combination with the shown source description builds an additional benefit of the WFA approach. The complete magnetic gradient information is available and the noise influences should not be correlated between the separate signal channels. The inputs of motion noise, which definitely show a correlation between the signal gradiometer channels, are correctable and will be discussed in more detail in Section 2.1.

The descriptions in the next chapter shows point-by-point investigations, where the combination of one dimensional analytical, algebraical and numerical methods is used to produce a set of solutions. These solutions are combined to get the resulting source description. The presented algorithm uses only the mathematical properties of single tensor component values and the whole tensor construction. Therefore, it differs elementary from investigations with multidimensional minimization approaches (Eichardt et al., 2009) or the analysis of ratios from calculated values (Oruç, 2010) along the measuring tracks. In principle, the detection of all the needed tensor components at one measuring point (together with a defined surrounding interval) offers the possibility of inversion only by using a few measuring points instead of whole lines or areas. Additionally, the usage of laminar survey data can ensure the enhancement of the inversion quality for example by using statistical approaches, which is discussed in Section 3.2.

In this study, firstly the properties and limitations of the basic algorithm will be shown and discussed. Subsequently, it is applied to a synthetic and a real measured data set on a test site which was especially prepared for system performance evaluation (Hauspurg, 2008). Further enhancements of the basic algorithm are shown and discussed.

The idea of the present work is not to explain all the details of the whole WFA algorithm itself. Instead, we want to describe how we have to change the basic WFA algorithm to get trustable inversion results. Hence, we give a compact overview about the basics of WFA published in detail in Frahm (1972) and Wynn (1998). Then, the focus is set to the mentioned two examples.

2. Theory

At the beginning we have to declare the basic assumptions of the algorithm. Firstly, the approximation of current absence causes a magnetic field $\vec{H}$ which is solenoidal and irrotational. Hence, a scalar potential can be introduced whose negative gradient is the magnetic field, see Eq. (1). Under these conditions, the used local gradient of Earth's magnetic field $\left(\nabla\circ\vec{B}\right)$ describes a symmetric ($G_{ij}=\partial B_i/\partial x_j=\partial x_i=G_{ji}$) and traceless ($G_{33}=-(G_{11}+G_{22})$) tensor of second order. So, the full tensor information is cached with 5 independent components instead of the universal number of nine, see Eq. (2).

Secondly, under the condition that the sensor to source distance is much larger than the lateral dimensions of a magnetic anomaly source it is described as dipole-like (Jackson, 1985).

For the terms of this work, the involved materials are approximated as "linear" because in practice their magnetization $\vec{M}$ shows almost a linear response to normal changes of naturally applied magnetic fields. The factor of proportionality is known as susceptibility χ. Technically, this can be written as

$$\vec{B}=\mu_0\left(\vec{H}+\vec{M}\right)=\mu_0(\chi+1)\cdot\vec{H}=\mu\cdot\vec{H}\,. \quad (3)$$

2.1. Requirements for the inversion algorithm

The FTMG system acquires the required full tensor magnetic gradient data and ensures the stationary conditions using integrated filters. Synchronization, scaling, decimation, balancing and geo-eferencing belong to a post processing step where the measured gradient values are combined to form the tensor (Fitzgerald et al., 2010; Stolz et al., 2006). At this stage, two dimensional maps of the investigated area are generated to illustrate the different gradient components and to select the various regions of interest. The acquired information exists in two-dimensional laminar grids (as shown in Fig. 1) or in a line based structure which can be used directly to apply the algorithm introduced in this work subsequently. The input of the calculations is a data set with five independent magnetic gradient components and the corresponding precise information of location and orientation. This is of particular importance since the algorithm output is always relative to the sensors. So, a very good knowledge of the measurement configuration advantages the success of the inversion. With the use of the orientation data the algorithm results can be transformed from the sensor's body coordinate system to a general and defined geo-referenced coordinate system, like north east down system (NED) for example.

Before discussing the inversion algorithm itself, the advantages of the full magnetic gradient tensor measurements should be illustrated. Theoretically, the main advantages of gradient measurements are a more detailed illustration and resolution of structural properties of the source and especially the suppression of the background field which is in general several orders of magnitude bigger than the field variations caused by anomalies of the hidden structures we want to detect (Munschy and Fleury, 2011; Schmidt and Clark, 2000). That is why, in our opinion, gradiometers are the most effective sensors to detect these very small anomalies during prospection measurements in motion. Another advantage of measuring the five independent magnetic gradient tensor components with separate sensors is the absence of correlation between the noise contributions. In comparison with possible error propagation by calculating the needed tensor values out of only one measurement (Lima and Weiss, 2009; Munschy and Fleury, 2011), the independent acquisition of several gradient values allows different error compensation approaches (for example with redundancy measurements, Schmidt and Clark, 2000). To get trustable inversion results it is necessary to use every possibility of the prevention of increasing systematical errors. In order to realize fast and effective field measurements the system includes additional SQUID magnetometers and the mentioned inertial unit. The magnetometers are used to eliminate the imperfection (like fabrication effects Zakosarenko et al., 1996) of the used real gradiometers. These corrections allow the compensation of motion noise caused by moving in Earth's magnetic field (Stolz et al., 2006).

After these preparations, a proper data input is realized to begin the inversion process. The proposed Wynn Frahm algorithm itself is a direct inversion approach separated into two processing steps. The Wynn part uses the correlation between signal slope and source position while the Frahm part solves the eigenvalue problem in the principal coordinate system which is described in detail in Frahm (1972) and Wynn (1998). The combination of both steps should allow