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Spatial attenuation rates of interfacial waves: Field and numerical tests of Sommerfeld theory using ground-penetrating radar pulses

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ABSTRACT

We tested the geometric amplitude attenuation rates predicted by classic Sommerfeld theory for horizontally polarized interfacial waves propagating over dielectric ground. We used ground-penetrating radar pulses, the brief time duration of which allowed different interfacial wave modes to separate. We tested rates in the intermediate range of tens of wavelengths, and for azimuthal and radial polarizations. For azimuthal polarization, a closed form solution predicts inverse range-squared rates, and for radial polarization, calculations suggest an inverse range exponent between 1 and 2. Over low loss frozen ground having a dielectric constant of 6.8 azimuthally polarized air waves centered at 46 MHz attenuated nearly in proportion to the square of range, as predicted, while the radial rate at 37 MHz was close to the 1.6 power of range, as generally expected. At 360–390 MHz, air wave rates were higher than expected and likely caused by scattering losses. Three D time domain modeling at 37 MHz confirmed the rate for azimuthal polarization and the qualitative difference in rates between the two polarizations, but the exponent may be about 26% too high for the radial case. Not readily extractable from Sommerfeld theory are rates for subsurface direct waves, for which our models show that both polarizations attenuate in proportion to the square of range after about 5 subsurface wavelengths. This suggests that geometric rates for all horizontally polarized subsurface interfacial waves spatially attenuate in proportion to range-squared in both intermediate and far field ranges, and so could be subtracted from actual rates to determine loss rates caused by intrinsic attenuation and scattering.

1. Introduction

Common midpoint (CMP) and single-sided moveout profiles are often used to obtain subsurface velocity information to aid interpretation of ground-penetrating radar (GPR) reflection profiles. In these methods GPR pulses that propagated along the surface and reflected from subsurface interfaces are continually recorded as a function of antenna separation. Often the direct interfacial waves are most prominent and last longest. The slopes of the time delay versus distance plots of these waves provide speeds from which ground dielectric constants (i.e. real part of the complex permittivity, ε') are obtained. Direct (non-reflected) azimuthally polarized interfacial subsurface waves (Fig. 1a) are commonly exploited to find ε' (Arcone and Delaney, 2003; Arcone et al., 2003a,b; Fisher et al., 1992), but radial polarization has been used as well (Arcone, 1984; Liu and Arcone, 2005; van der Kruk et al., 2007). Dispersive modes guided within a near-surface layer have also been used to derive ε' (Arcone et al., 2003a,b; van der Kruk et al., 2007). Additional information regarding ground loss factors such as conductivity, dielectric loss and scattering is contained in the attenuation rates of these waves. In order to extract this information however, one must know the geometric spreading loss rate so that it can be subtracted from the calculated rates. Here, we use GPR pulses experimentally at a real field site, and numerically with 3-D modeling computations to test geometric attenuation rates predicted by well established monochromatic theory.

By interfacial, we mean waves propagating on either side of an interface between materials of different electrical properties. We refrain from the use of the term “surface wave” because it suggests propagation along only the top of a surface and in air. In a cylindrical coordinate system the horizontal components are the azimuthal electric field, E_ϕ (where ϕ is the azimuthal polar coordinate) and the radial electric field, E_ρ (where ρ is the radial coordinate) (Fig. 1a). Along the direction normal to the antenna axis E_ϕ is also known as the transverse electric, or broadside field, while in the direction along the axis E_ρ is also known as the endfire field. Over smoothly layered ground both components will propagate as direct interfacial waves in air and in the ground, the latter of which are accompanied by subsurface interfacial reflections.

The geometric spreading rates have been well formulated for both E_ϕ and E_ρ, but to our knowledge never tested experimentally or numerically. Sommerfeld (1926) first developed the theory for conductive earth at less than about 1 MHz. Reformulations (Banos,

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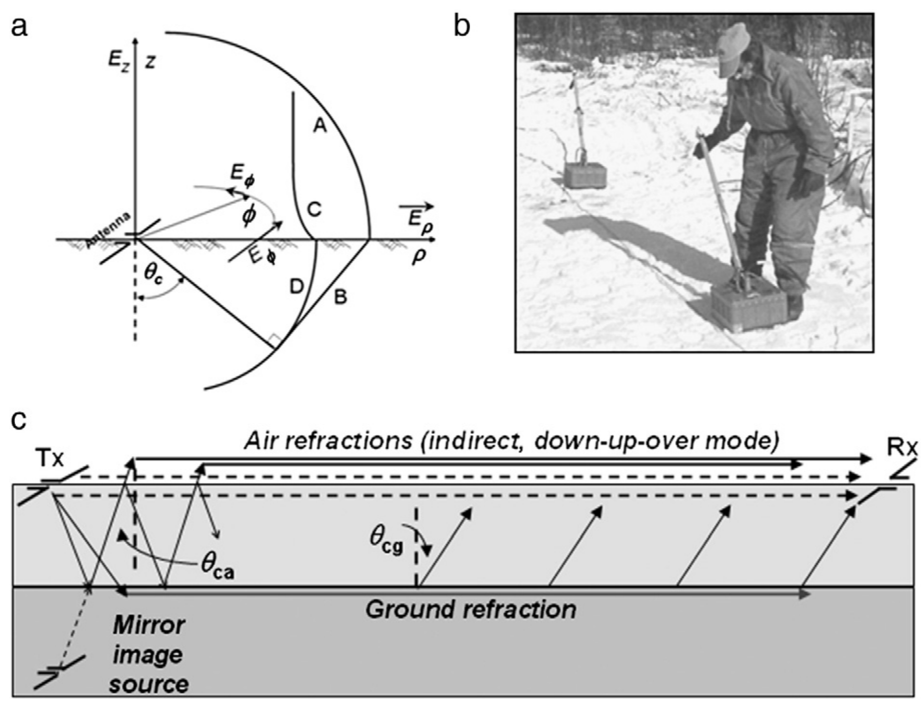

Fig. 1. (a) Representation of waves launched by an antenna lying on homogeneous ground (after Annan, 1973) described by a cylindrical coordinate system; (b) file photograph of a moveout profile being performed with "400-MHz" antenna units; and (c) diagram of interfacial events. In (a) A and B are spherical waves, and C and D are matching evanescent and head waves, respectively. The phase front of D propagates at the critical angle, $\theta_c = \sin^{-1}(1/n)$, with respect to vertical. E_ϕ and E_ρ are the broadside and endfire field components, respectively. In (b) the transmitter antenna of one unit and the receiver of the other unit are used. In (c) long dashed arrows represent direct air and subsurface waves, and solid arrows represent air and ground refractions propagating from a transmitter (Tx) to a receiver antenna (Rx). The air refractions may be multiple, launched by successive subsurface reflections. We used a mirror image source for Tx to simulate the down-up-over mode in our numerical integration of the Sommerfeld solutions.

1966; Norton, 1936, 1937) have been applied at higher frequencies to homogeneous and layered earth with complex permittivity, ε^* (Annan, 1973; King et al., 1980; Wait, 1951), and for antennas at any height above or below the surface. On the surface of dielectric earth and in the direction normal to the antenna axis, the solutions are simple, exact and well known theoretically for E_ϕ (Annan, 1973; Wait, 1951), but only approximations to integral solutions exist for E_ρ in the direction parallel to the antenna axis (Banos, 1966; King et al., 1980).

Following Banos (1966) and King et al. (1980), above about 10 MHz geometric attenuation rates within several ranges, R, from a transmitting antenna may be characterized by a $1/R^\alpha$ function. Within only one to two free space wavelengths (λ_o; $\lambda = \lambda_o/\sqrt{\varepsilon'}$ is the in situ wavelength), defined as the "near range" (NR), E_ϕ and E_ρ are predicted to transition from $\alpha = 3$ to $\alpha \leq 2$ dependency. Beyond this distance E_ϕ is predicted to follow $\alpha = 2$ permanently, as described mathematically in the next section. Within an intermediate range (IR) of a few to possibly several tens of free space wavelengths, E_ρ is predicted to exhibit a lesser α, possibly near unity, after which it reaches $\alpha = 2$ dependency in the "far range" (FR), also known as the Asymptotic Range (Banos, 1966; in contrast, $1/R$ dependency begins in the "far field" for free space or "body" waves). Similar dependencies might be expected for the subsurface direct wave (D in Fig. 1a), but were not treated by Banos (1966); lossy ground would provide only NR solutions. We are interested in the IR because most GPR systems have cable lengths that limit antenna separation to about 100 m; only greater ranges have been investigated numerically (King et al., 1980).

Our objective was to determine attenuation rates for horizontal polarization and homogeneous dielectric ground in the IR. We measured and calculated amplitudes of GPR pulses, which we recorded in field (Fig. 1b), and in 3-D numerical simulations of moveout profiles. The brief time duration of our pulses allowed us to separate direct air waves (in the field) and direct subsurface waves (numerically) from other, indirect events generated by subsurface interfacial and body waves that propagated along or had returned to, the surface, respectively. We used pulse bandwidths centered near 37–71 MHz and 360–390 MHz, which were determined by the available antennas and the ground impedance loading they experienced. In the field we focused on air waves because only they were isolated from interference and experienced no absorptive attenuation from ground electrical properties. We measured the amplitude of the leading cycles, one case of which was an indirect wave returned to the surface (explained in the next section). We performed our field experiments at a two-layer site for which the top layer was sufficiently thick, frozen and smooth, to allow direct and indirect interfacial air waves to be identified and measured, thus providing a proxy for low loss homogeneous ground. The span of frequencies, the apparently flat surface and low loss properties allowed us to assess the possible influence of scattering loss. Numerically, we generated both air and subsurface interfacial 3-D pulsed waves for homogeneous ground, which allowed us to isolate the subsurface waves. We chose parameters to match our field situation. We used a pseudospectral finite element method but only at 37 MHz; we appeared to encounter stability difficulties at 370 MHz.

We compared our air wave rates against numerical integrations of the Sommerfeld theory for both horizontal polarizations. The single frequency nature of this theory does not permit integral calculations that separate various wave modes and so the direct air wave is usually computed because it quickly separates from all other modes. Given that $\alpha = 2.00$ is predicted for azimuthal polarization by closed-form solution, we used deviations from this rate to estimate computational errors or additional, unwanted losses in all cases. In particular, any deviation of the Sommerfeld integrations from $\alpha = 2.00$ for E_ϕ would establish a likely error for computations of E_ρ. Although we used an effective α to describe the rates, the actual dependency is likely slightly more complicated. In the field we calculated a ground $\varepsilon' = 6.8$, for which a significant effect upon the rate for E_ρ is predicted by Sommerfeld theory below about 100 MHz. Therefore, we used this value of ε' for our numerical calculations as well. As ε' increases the