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Retrieval of the physical properties of an anelastic solid half space from seismic data

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ABSTRACT

In recent years, due to the rapid development of computation hard- and software, time domain full-wave inversion, which makes use of all the information in the seismograms without appealing to linearization, has become a plausible candidate for the retrieval of the physical parameters of the earth's substratum. Retrieving a large number of parameters (the usual case in a layered substratum comprising various materials, some of which are porous) at one time is a formidable task, so full-wave inversion often seeks to retrieve only a subset of these unknowns, with the remaining parameters, the priors, considered to be known and constant, or sequentially updated, during the inversion. A known prior means that its value has been obtained by other means (e.g., in situ or laboratory measurement) or simply guessed (hopefully, with a reasonable degree of confidence). The uncertainty of the value of the priors, like that of data noise, and the inadequacy of the theoretical/numerical model employed to mimick the seismic data during the inversion, is a source of retrieval error. We show, on the example of a homogeneous, isotropic, anelastic half-plane substratum configuration, characterized by five parameters: density, P and S wavespeeds and P and S quality factors, when a perfectly-adequate theoretical/numerical model is employed during the inversion and the data is free of noise, that the retrieval error can be very large for a given parameter, even when the prior uncertainty of another single parameter is very small. Furthermore, the employment of other load and response polarization data and/or multi-offset data, as well as other choices of the to-be-retrieved parameters, are shown, on specific examples, not to systematically improve(they may even reduce) the accuracy of the retrievals when the prior uncertainty is relatively-large. These findings, relative to the recovery, via an exact retrieval model processing noiseless data obtained in one of the simplest geophysical configurations, of a single parameter at a time with a single uncertain prior, raises the question of the confidence that can be placed in geophysical parameter retrievals: 1) when more than one parameters are retrieved at a time, and/or 2) when more than one prior are affected by uncertainties during a given inversion, and/or 3) when the model employed to mimick the data during the inversion is inadequate, 4) when the data is affected by noise or measurement errors, and 5) when the parameter retrieval is carried out in more realistic configurations.

1. General introduction

An important branch of activity in geophysics is concerned with obtaining structural (tomographic) images of the subsurface and estimating the physical properties of the different layers. Although structural details can be obtained by the migration of (either man-made or natural) seismic data (a process that employs only a small amount of the information in the seismogram), retrieving the physical properties of the geological formations (as is important, for instance, in hydrocarbon exploration and prediction of seismological site effects) requires a seismic inversion method (Forbriger, 2003; Sacks and Symes, 1987; Tarantola, 1986). Travel-time inversion (TTI; which also uses only a small amount of the information in the seismogram, i.e., the picked travel times) (Bodet, 2005; Foti et al., 2009; Luo and Schuster, 1991; Pereyra et al., 1980; Xia et al., 1999) or full waveform inversion (FWI; which employs most or all of the information in the seismogram and does not require fastidious and sometimes ambiguous travel-time picking) techniques have been employed either separately (De Barros et al., 2010; Delprat-Jannaud and Lailly, 2005; Dupuy, 2011; Jeong et al., 2012; Mora, 1987; Pratt, 1999; Sebaa et al., 2006; Song et al., 1995; Sun and McMechan, 1992; Virieux and Operto, 2009) or together with TTI (Buchanan et al., 2011; Chotiros, 2002; Dessa and Pascal, 2003; Korenaga et al., 1997) to retrieve wave-mechanical parameters of the substratum, such as Poisson's ratio, density, quality factors and body wave velocities, by minimizing the differences between real (either measured or synthetic) and trial seismic data generated by a more-or-less rigorous wave propagation model together with a subsurface model of supposedly-appropriate structure and composition.

An annoying problem in the characterization of the substratum is the large number of unknown parameters; this is termed the 'curse of dimensionality' in (Curtis and Lomax, 2001). A typical inversion

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algorithm assumes that most of the parameters are known a priori (these parameters are termed *priors* (Scales and Tenorioz, 2001)); in Sebaa et al. (2006) six out of eleven of the porous medium parameters are treated as fixed priors, so that the inversions are carried out only for the remaining five parameters, in (Aoi et al., 1995) the basin velocities and details of the solicitation are treated as fixed priors during the determination of the basin shape, and in (Burstedde and Ghattas, 2009) and (Kolb et al., 1986) the density is treated as a known, fixed prior during the retrieval of the velocity (or rigidity) distribution in a 1D acoustic (or elastic) medium. Another option is to assume that certain priors are related in some manner (such as in a Poisson solid) and the remaining parameters are sought-for in a rather large parameter search space.

Similar remarks apply to the to-be-retrieved parameters. The initial estimation of the latter is often far-removed from reality, but can be refined by the reduction of the size of the parameter search space, wherein the trial data is again compared to the real data to obtain a second estimation of the unknown parameters. This iterative process, with the set of priors conserving their initial values, is repeated as many times as necessary to obtain some pre-assigned estimation accuracy.

Another approach to multiparameter retrieval, which seeks to alleviate somewhat the 'curse of dimensionality', proceeds in a so-called *sequential* fashion (De Barros et al., 2010; Forbriger, 2003; Kormendi and Dietrich, 1991). Suppose the number of parameters is two. In the first step, p_1 is treated as a known prior (the notion of 'known' can have many meanings ranging from 'measured' (e.g., some of the Biot parameters in Buchanan et al., 2011) to 'guessed' (e.g., the quality factors in (Forbriger, 2003)) and the inversion is carried out only for p_2. In the second step, the retrieved value of p_2 is treated as a prior and the inversion is carried out to retrieve p_1. In the third step, the newly-retrieved value of p_1 is treated as a prior and the inversion is carried out for p_2. This procedure could go on indefinitely, but usually stops at an early level. First attempts at a critical analysis of this technique were carried out in (Le Marrec et al., 2006), but gave rise to no convincing conclusion. It seems reasonable to expect that this iterative procedure will diverge if at the initial stage, the inversion already leads to highly-erroneous results for p_2 due to relatively-large uncertainty in the prior p_1.

Inversion algorithms such as the procedures described above, are not easy to implement, in that they usually demand manual surveillance and intervention. Moreover, at each step, an interpretation problem can arise, as attested by the fact that the algorithm can give rise to three types of results (Wirgin, 2004): 1) either the retrievals are obtained without any apparent difficulty, i.e., at most iteration steps, the discrepancy function resembles a single paraboloid (in the parameter search space) with a well-defined minimum (Sebaa et al., 2006), 2) the discrepancy function does not exhibit a paraboloid type of behavior in the successive parameter search spaces so that no estimation can be made of the sought-for parameters, or 3) the discrepancy functions exhibit several local minima (Buchanan et al., 2011; Ogam et al., 2001, 2002) so that a decision has to be made as to which minimum is the most appropriate.

It has been shown (Buchanan et al., 2002; Groby et al., 2011; Ogam et al., 2001; Wirgin, 2004) on some rather simple examples of inverse problems, that: a) the most favorable situation (case 1) occurs when the retrieval model gives rise to data that is as close as possible to the real data throughout the parameter search space, and b) the least favorable situations (cases 2) and 3)) occur when the retrieval model gives rise to data that is significantly different from the real data in a part of, or throughout, the parameter search space. Interestingly enough, efforts (Buchanan et al., 2000, 2002; Scotti and Wirgin, 1995; Wirgin and Scotti, 1996) have been made to deliberately employ a retrieval model that is different, by its theoretical and numerical ingredients, from the model used to obtain synthetic data, notably to avoid what in (Colton and Kress, 1992) is termed a 'trivial solution' to the inverse problem. Avoidance of 'trivial solutions' can also be obtained by artificially introducing noise into the synthetic data (Aoi et al., 1995; Buchanan et al., 2002). However, such efforts are generally deemed not necessary, or the issue of 'trivial solutions' is simply sidetracked (Aoi et al., 1995; De Barros et al., 2010; Sebaa et al., 2006) by the use of the same theoretical/numerical scheme for the simulation and the retrieval of the unknown parameters.

The present investigation addresses the issue of the effect of prior uncertainty on the accuracy of parameter retrieval. This type of study was initiated in (Aoi et al., 1995; Buchanan et al., 2002, 2011; Chotiros, 2002; De Barros et al., 2010; Dupuy, 2011; Jeong et al., 2012; Scotti and Wirgin, 2004) and usually relies on synthetic data that is created with the same theoretical/numerical model as the one employed for the retrievals. Although this seems to imply that a 'trivial solution' is generated, it can be argued that prior uncertainty has an effect somewhat equivalent to introducing a difference between the two models.

The chosen geophysical configuration herein is a homogeneous, isotropic, *anelastic* half-space (i.e., the underground or substratum) solicited by a transient load on its flat boundary (i.e., the ground) which generates seismic waves acquired by receivers located on this boundary. Five reasons explain this choice.

1 Although it is unrealistic in the context of natural resource exploration, it has been employed to examine various aspects of a variety of other applied geophysical problems (de Barros et al., 2010; Goldman et al., 1996; Hurley and Spicer, 2004; Kozhevnikov and Antonov, 2008; Muijs et al., 2002; Wu et al., 2001), notably to furnish a macromodel (reference) medium for the background Green's function employed in tomographical imaging (via Born or Rytov schemes) of a layer-like or bounded heterogeneity in an otherwise macroscopically-homogeneous underground (Cox, 1991; Gelis, 2005; Gelis et al., 2007; Operto et al., 2004; Ribodetti and Virieux, 1998; Van der Made, 1988). Note that in most of the latter-cited references, the background medium is assumed to be known beforehand, whereas, in field experiments it is usually as unknown as the layer-like or bounded heterogenities contained therein, a fact that explains why the physical parameters of the background must be retrieved by inversion before proceeding to the imaging of the heterogeneities.
2 The homogeneous, isotropic half space configuration is often used (and called the Okada model Okada, 1985) to estimate earthquake source parameters from geodetic data (Amoruso et al., 2004; Feigl, 2002).
3 It can also be of interest to seismologists concerned with predicting earthquake site effects, since the concept of amplification of ground motion is often related (Bard and Tucker, 1985; Boore, 1972; Bouchon, 1973; Fäh et al., 1994; Geli et al., 1988; Kawase and Aki, 1989) to a ratio of motion at a point on a ground that is not flat (i.e., on a hill or in a basin) and/or overlies relatively soft layers (i.e., an alluvial basin or a weathered hill) to the motion at a point on flat ground overlying hard rock, and in order for these ratios to be meaningful, the mechanical parameters of the rock (overlain by flat ground) have to be identified beforehand.
4 The use of the geophysically-unrealistic homogeneous, isotropic half plane model provides one of the simplest illustrations (as shown hereafter) of the deleterious effect of prior uncertainty on subsurface parameter retrieval error, and these effects are surely of the same order or even of greater order in more complicated, more realistic (e.g., vertically or laterally-inhomogeneous) subsurface configurations as arise in field experiments (Delprat-Jannaud and Lailly, 2005; Van Vossen et al., 2004).
5 Seminal contributions to nonlinear FWI such as (Kolb et al., 1986; Tarantola, 1984) relied on the more drastic elements of unreality constituted by the acoustic approximation, and/or infinite quality factors (Tarantola, 1986).

Although inversion necessarily appeals to a theoretical/numerical apparatus for solving the forward problem, we shall only briefly recall