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Determination of the maximum-depth to potential field sources by a maximum structural index method

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ABSTRACT

A simple and fast determination of the limiting depth to the sources may represent a significant help to the data interpretation. To this end we explore the possibility of determining those source parameters shared by all the classes of models fitting the data. One approach is to determine the maximum depth-to-source compatible with the measured data, by using for example the well-known Bott–Smith rules. These rules involve only the knowledge of the field and its horizontal gradient maxima, and are independent from the density contrast.

Thanks to the direct relationship between structural index and depth to sources we work out a simple and fast strategy to obtain the maximum depth by using the semi-automated methods, such as Euler deconvolution or depth-from-extreme-points method (DEXP).

The proposed method consists in estimating the maximum depth as the one obtained for the highest allowable value of the structural index (N_{max}). N_{max} may be easily determined, since it depends only on the dimensionality of the problem (2D/3D) and on the nature of the analyzed field (e.g., gravity field or magnetic field). We tested our approach on synthetic models against the results obtained by the classical Bott–Smith formulas and the results are in fact very similar, confirming the validity of this method. However, while Bott–Smith formulas are restricted to the gravity field only, our method is applicable also to the magnetic field and to any derivative of the gravity and magnetic field. Our method yields a useful criterion to assess the source model based on the $(\partial f/\partial x)_{\max}/f_{\max}$ ratio.

The usefulness of the method in real cases is demonstrated for a salt wall in the Mississippi basin, where the estimation of the maximum depth agrees with the seismic information.

1. Introduction

Inverse potential field problems are inherently difficult to solve because they are ill-posed and do not have a unique solution. Moreover, the calculated solution may be extremely sensitive to errors. We pose the forward problem as:

$$f(r) = \int_V A(\mathbf{r}, \mathbf{r}_0) M(\mathbf{r}_0) d\mathbf{r}_0{}^3 \tag{1}$$

where f is the field, measured outside the source volume V, and the unknown source distribution is described by a continuous function M inside V, $\mathbf{r_0}$ denotes the position of a point inside the source volume, and $\mathbf{r}$ denotes an observation point outside V. The function A is the Green's function for the gravitational or magnetic sources.

However, Eq. (1) is a first-kind Fredholm equation, known to be an ill-posed problem. Even if we had access to noise-free and continuous field data, we would face an ambiguity problem: by Green's third identity, any potential field in a sub-region can be reproduced by an infinite variety of surface distributions. Moreover, any source distribution producing a null field, belonging to the so-called annihilator (Parker, 1977), cannot be determined from the data. Other kinds of ambiguity may also be mentioned: a sampling ambiguity occurring because discrete data sets may be insufficient to completely represent the continuous field from the source; algebraic ambiguity, occurring in inverse theory because the source discretization usually leads to a system with more unknowns than data; noise "ambiguity", referring to components of the solution that cannot be recovered due to errors and noise, i.e., components that are practically undetermined.

To face these ambiguities, some sort of incorporation of a priori knowledge is needed, in order to compute unique solutions. The a priori information can be supplied in many ways, as to minimize the density model weighted norm with respect to some reference model (Green, 1975); constraining a linear least-squares problem for upper and lower density bounds and for a density monotonically increasing with depth (Fisher and Howard, 1980); searching for compact solutions (Last and Kubik, 1983; Pilkington, 2009; Zhdanov, 2002) or for solutions that are smooth in some sense, e.g., as measured by a (weighted) norm of the solution itself and/or some of its derivatives. In this sense Li and Oldenburg (1996) obtained depth

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resolution by incorporating a suitable depth weighting function in the regularization formulation.

An entirely different approach to face the ambiguity is to determine which model parameters are common to all models fitting the data, or at least to wide classes of all the models. For instance the total excess mass of the buried body is a parameter shared by all the possible models and may be determined uniquely if the anomaly is enough isolated and well sampled (Grant and West, l965).

A similar reasoning occurs for the boundaries on density and depth expected for the whole set of the sources. They may be uniquely specified even when the data are incomplete, according to the maximum-depth rules (e.g., Smith, 1959) or by determining the class of "ideal" bodies which achieve the extreme values of depth or density (Parker, 1974). In particular, the maximum-depth rules allow a rapid estimation of the maximum source depth from the anomaly and its gradient at specific points and by assuming a maximum for the density by geological or other information. Such estimates can be obtained from anomalies generated by isolated sources. Among several formulas of this kind (e.g., Smith, 1959) we consider of great interest those developed by Bott and Smith (1958) for the gravity field:

$$z_0 \leq \frac{48\sqrt{5}}{125}\frac{f_{max}}{\left|\frac{\partial f}{\partial x}\right|_{max}} \tag{2}$$

for the 3D case, and

$$z_0 \leq \frac{3\sqrt{3}}{8}\frac{f_{max}}{\left|\frac{\partial f}{\partial x}\right|_{max}} \tag{3}$$

for the 2D case.

The important feature of these formulas is that they depend uniquely on the values of the field and of its horizontal gradient, so that no other restrictions are placed on the magnitude or variation of the density, or on the shape of the body. According to Bott and Smith (1958) these limiting estimates refer to the topmost part of the source. We could however obtain biased solutions from Eqs. (2) and (3) in presence of regional fields (affecting the value of f_{max}), so that a preliminary detrending is highly recommended.

We will describe in this paper how semi-automatic methods based on the properties of the homogeneous fields, such as the Euler deconvolution (e.g., Nabighian and Hansen, 2001; Reid et al., 1990) and the "Depth from EXtreme Points" (DEXP) method (Fedi, 2007; Fedi and Pilkington, 2012), may be used as an alternative and stable approach to define the maximum depth to the sources. In this paper we will use mainly the DEXP method, which is characterized by a composite upward continuation–differentiation operator acting as a band-pass operator (Fedi et al., 2009). This feature makes the DEXP method very stable vs. the noise, even using a high-order vertical or horizontal derivative of the field. We will show in the next section how the maximum depth may be easily determined with DEXP method by using the highest allowable value of the structural index N.

2. Determining the maximum depth by the DEXP method

2.1. The highest allowable value of the structural index

As well known, the value of the structural index N varies according to the type of the field and to the source type (e.g., Reid et al., 1990; Hsu, 2002). The highest allowable value N_{max} is in practice determined by the nature of the analyzed field (gravity field or magnetic field) and by the dimensionality of the problem (2D or 3D). For instance, in the 3D gravity case the allowed values for N are the integers [−1, 0, 1, 2] so that $N_{max}=2$. In the 3D magnetic case the allowed values for N are instead the integers [0, 1, 2, 3] so that $N_{max}=3$.

In the 2D case, N_{max} is equal to 1 and 2, respectively for the gravity and magnetic case, because no 3D ideal sources (spheres) are allowed. Finally, as shown in Table 1, N_{max} is increased by the differentiation order p whenever a vertical or horizontal derivative of gravity and magnetic fields is considered.

The selection of the maximum value of the allowable structural index method is thus very simple (Table 1) and our method is fast to apply, as it will be shown in the following sections.

2.2. The DEXP transformation

The DEXP transformation (Fedi, 2007) is a simple transformation applied to the p-order derivative of the potential field f, which we indicate as f_p:

$$\Omega_p = z^{N_p/2} f_p \tag{4}$$

where z is the altitude and:

$$N_p = N + p. \tag{5}$$

The numerical implementation of the method uses upward continuation and vertical differentiation for the computation of f_p.

The DEXP transformation enjoys useful properties (Fedi, 2007), the main being that the vertical position of its maxima $\hat{z}$ yields the depth to the source z_0, simply as $z_0=-\hat{z}$. This rule applies to homogeneous fields. For instance, in the magnetic case, homogeneous fields are generated by ideal sources such as the point dipole ($N=3$), the infinite horizontal and the semi-infinite vertical cylinders ($N=2$), the dike and sill ($N=1$) and the contact ($N=0$).

From Eq. (4) we argue that the DEXP transformation needs N_p to be either assumed or estimated in advance. This may be made in several ways, from the scaling function (Fedi, 2007; Florio et al., 2009) or also using Euler deconvolution algorithms (e.g., Nabighian and Hansen, 2001). However, we are not really interested here in this task. We want in fact to use a different property of the DEXP transformation, allowing us to determine the maximum depth to the source. This property is the linear relationship between N and z_0, clearly shown by several authors (e.g., Barbosa et al., 1999) in the case of Euler deconvolution. Thanks to this property, the estimated depth to the source will increase as N increases and the maximum depth will then be reached at the maximum allowable N (N_{max}). So we can obtain the maximum depth by the maxima of Ω_p according to:

$$\Omega_p = z^{N_{\max}/2} f_p \tag{6}$$

2.3. Purely 3D and 2D sources

In this section we will show how to obtain a maximum depth estimate by using the DEXP method. We will test the maximum-depth consistency of the DEXP estimate (Eq. (6)) by comparison with the

Table 1
$N_{\max}$ for potential fields.

Dimensionality	Gravity field	p-order derivative of Gravity field	Magnetic field	p-order derivative of Magnetic field	Corresponding model
3D	2	$2+p$	3	$3+p$	Sphere
2D	1	$1+p$	2	$2+p$	Infinite horizontal cylinder, infinite vertical cylinder