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Source-independent elastic waveform inversion using a logarithmic wavefield

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ABSTRACT

The logarithmic waveform inversion has been widely developed and applied to some synthetic and real data. In most logarithmic waveform inversion algorithms, the subsurface velocities are updated along with the source estimation. To avoid estimating the source wavelet in the logarithmic waveform inversion, we developed a source-independent logarithmic waveform inversion algorithm. In this inversion algorithm, we first normalize the wavefields with the reference wavefield to remove the source wavelet, and then take the logarithm of the normalized wavefields. Based on the properties of the logarithm, we define three types of misfit functions using the following methods: combination of amplitude and phase, amplitude-only, and phase-only. In the inversion, the gradient is computed using the back-propagation formula without directly calculating the Jacobian matrix. We apply our algorithm to noise-free and noise-added synthetic data generated for the modified version of elastic Marmousi2 model, and compare the results with those of the source-estimation logarithmic waveform inversion. For the noise-free data, the source-independent algorithms yield velocity models close to true velocity models. For random-noise data, the source-estimation logarithmic waveform inversion yields better results than the source-independent method, whereas for coherent-noise data, the results are reversed. Numerical results show that the source-independent and source-estimation logarithmic waveform inversion methods have their own merits for random- and coherent-noise data.

1. Introduction

Although, seismic waveform inversion is a promising method for providing detailed subsurface velocity information, there are many obstacles, such as the local minima problem, the absence of low frequencies, etc., which prevent successful inversion. Numerous studies have been devoted to develop a robust waveform inversion algorithm. Among them, Shin and Min (2006) proposed the logarithmic waveform inversion in the frequency domain, and it has been widely applied in both the frequency and Laplace domains (Bednar et al., 2007; Pyun et al., 2007; Shin and Cha, 2008, 2009; Shin et al., 2007; Shin et al., 2010). Taking the logarithm of a wavefield separates the amplitude (real part) and phase (imaginary part) of the Fourier transformed wavefield, where the amplitude and phase are related to the energy and kinematic properties of the wavefield, respectively. If we use only the phase information (imaginary part) of the logarithmic wavefield, its inversion is very similar to travel-time tomography (Min and Shin, 2006). Shin et al. (2007) described the feasibility of the logarithmic waveform inversion and suggested that it is tomographic in the early stage of the inversion and more dependent on amplitude differences in the later stages.

All of the former waveform inversion algorithms, that use the logarithmic wavefields, estimated the source wavelet along with the model parameters (source-estimation logarithmic waveform inversion: SELWI). Although source wavelet information is necessary for successful waveform inversion, it is not easy to estimate the exact source wavelet when the exact subsurface parameters are unknown (Pratt, 1999). To avoid source estimation, Lee and Kim (2003), Zhou and Greenhalgh (2003), Choi et al. (2005), and Xu et al. (2006) developed the source-independent waveform inversion algorithms (SIWI). All of these researchers normalized the wavefields by the reference wavefield to remove the effects of the source wavelet and used the normalized wavefields to construct the misfit function. By doing so, they succeeded in recovering subsurface parameters without considering source wavelet information for the synthetic data examples.

In this study, we develop a source-independent logarithmic waveform inversion algorithm and investigate its robustness for noisy data. For the source-independent logarithmic waveform inversion (SILWI), we first normalize the wavefield by the reference wavefield, and then take the logarithm of the normalized wavefield. As the reference wavefield, we consider the nearest-offset trace. Taking the logarithm of the frequency-domain wavefield allows us to separate the amplitude and phase and to develop three kinds of source-independent logarithmic waveform inversion using a) the amplitude-only (SILWI-A), b) the phase-only (SILWI-P), and c) the both (SILWI-B). To investigate the robustness and accuracy of our algorithms, we compare our algorithms with the source-estimation logarithmic waveform inversion (SELWI) using the amplitude-only (SELWI-A), the phase-only (SELWI-P), and the both (SELWI-B). The misfit function is constructed by using the l_2

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norm of the differences between logarithms of the normalized modeled and observed wavefields in the frequency domain. The modeled wavefields are computed by the finite-element method. The gradient of the misfit function is computed on the basis of the adjoint state of modeling operator (Cao et al., 1990; Choi et al., 2008a; Gauthier et al., 1986; Kolb et al., 1986; Lailly, 1983; Plessix, 2006; Pratt et al., 1998; Shin and Min, 2006; Tarantola, 1984; Zhou et al., 1995).

In the following sections, we will first introduce the three types of misfit functions for the source-independent logarithmic waveform inversion and then provide the expression of the gradient direction for the three types using the back-propagation algorithm. Next, we will provide some numerical examples obtained by applying our source-independent logarithmic waveform inversion to the noise-free, random noise-included, and coherent noise-included synthetic data for the modified version of the elastic Marmousi2 model (Martin et al., 2002).

2. Theory

2.1. Misfit functions

Because recorded seismic data are expressed by the convolution of impulse response with the source wavelet in seismic exploration, they can be expressed by the multiplication of the impulse response and source wavelet in the frequency domain as follows:

$$d_j = g_j s, \tag{1}$$

where d_j is the observed wavefield at the jth receiver, g_j is the impulse response (Green function) at the jth receiver, and s is the source wavelet in the frequency domain. Normalizing the wavefield in Eq. (1) with respect to an arbitrarily chosen reference wavefield (indicated by the *ref* subscript) in the observed data gives

$$\frac{d_j}{d_{\text{ref}}} = \frac{g_j s}{g_{\text{ref}} s} = \frac{g_j}{g_{\text{ref}}}. \tag{2}$$

In Eq. (2), the source wavelets in both the denominator and the numerator are canceled out and only the impulse responses remain. Similarly, for the modeled data, the source wavelets are canceled out by dividing the modeled wavefield by a reference modeled wavefield. The reference wavefield can be either the nearest-offset trace or an average of traces. In this study, we adopt the nearest-offset trace for the reference wavefield for both the observed and modeled data.

Since the Fourier-transformed wavefield has a complex value ($Ae^{i\theta}$), taking the logarithm of normalized wavefield in Eq. (2) gives

$$\ln\left(\frac{d_j}{d_{\text{ref}}}\right) = \ln\left(\frac{A_j^{\text{d}}}{A_{\text{ref}}^{\text{d}}}\right) + i\left(\theta_j^{\text{d}} - \theta_{\text{ref}}^{\text{d}}\right), \tag{3}$$

where A is the amplitude, θ is the phase, and the superscript d represents the observed wavefield. In Eq. (3), the real part is the logarithm of amplitude ratio and the imaginary part is the phase difference. As the source wavelet is removed in Eq. (2), the amplitude and the phase of the source wavelet is removed:

$$\ln\left(\frac{A_j^{\text{d}}}{A_{\text{ref}}^{\text{d}}}\right) = \ln\left(\frac{A_j^{\text{g}} A^{\text{s}}}{A_{\text{ref}}^{\text{g}} A^{\text{s}}}\right) = \ln\left(\frac{A_j^{\text{g}}}{A_{\text{ref}}^{\text{g}}}\right), \tag{4}$$

$$\theta_j^{\text{d}} - \theta_{\text{ref}}^{\text{d}} = \left(\theta_j^{\text{g}} - \theta^{\text{s}}\right) - \left(\theta_{\text{ref}}^{\text{g}} - \theta^{\text{s}}\right) = \theta_j^{\text{g}} - \theta_{\text{ref}}^{\text{g}}, \tag{5}$$

where the superscripts g and s represent impulse response and source wavelet, respectively. Based on Eq. (3), we can construct three types of misfit functions for the source-independent logarithmic waveform inversion using the l_2-norm of residuals between the observed and the modeled wavefields. The three types of misfit function are as follows: the combination of amplitude and phase, Eq. (6); the amplitude-only, Eq. (7); and the phase-only, Eq. (8). The following equations are used for the misfit function:

$$E_1 = \sum_j \frac{1}{2}\left[\ln\left(\frac{u_j}{u_{\text{ref}}}\right) - \ln\left(\frac{d_j}{d_{\text{ref}}}\right)\right] \cdot \left[\ln\left(\frac{u_j}{u_{\text{ref}}}\right) - \ln\left(\frac{d_j}{d_{\text{ref}}}\right)\right]^*, \tag{6}$$

$$E_2 = \sum_j \frac{1}{2}\left[\ln\left(\frac{A_j^u}{A_{\text{ref}}^u}\right) - \ln\left(\frac{A_j^d}{A_{\text{ref}}^d}\right)\right]^2, \tag{7}$$

$$E_3 = \sum_j \frac{1}{2}\left[\left(\theta_j^u - \theta_{\text{ref}}^u\right) - \left(\theta_j^d - \theta_{\text{ref}}^d\right)\right]^2, \tag{8}$$

where u_j is the modeled wavefield at the jth receiver, u_{ref} is the modeled wavefield at the reference receiver, u superscript indicates the modeled wavefields, and * superscript represents the complex conjugate. The amplitude-only inversion is based on the assumption that the phase of the modeled data is the same as that of the observed data (Pyun et al., 2007).

2.2. Gradient directions using the back-propagation algorithm

The gradient of the misfit function is obtained by taking the partial derivative of the misfit function with respect to the model parameters. As a result, the gradient is composed of the partial derivative of the modeled data with respect to the model parameters. The partial derivative of the modeled wavefield ($u_j = A_j e^{i\theta_j}$) with respect to the kth model parameter p_k is given by the following equation:

$$\frac{\partial u_j}{\partial p_k} = \frac{\partial A_j}{\partial p_k} e^{i\theta_j} + iA_j e^{i\theta_j} \frac{\partial \theta_j}{\partial p_k}. \tag{9}$$

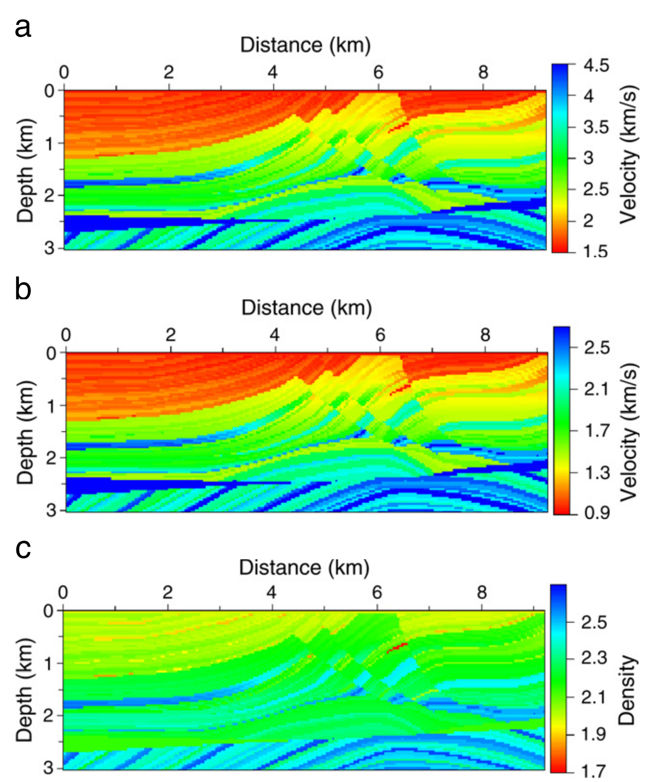

Fig. 1. True (a) P- and (b) S-wave velocities, and (c) density of the modified version of the Marmousi2 elastic model used for generating the synthetic data for waveform inversion.