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Combined straightforward inversion of resistivity and induced polarization (time-domain) sounding data

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ABSTRACT

It is proposed that the Straightforward Inversion Scheme (SIS) developed by the authors for 1D inversion of resistivity sounding and magneto-telluric sounding data can also be used in similar fashion for time-domain induced polarization sounding data. The necessary formulations based on dynamic dipole theory are presented. It is shown that by using induced polarization potential, measured at the instant when steady state current is switched off, an equation can be developed for apparent 'chargeability–resistivity' which is similar to the one for apparent resistivity. The two data sets of apparent resistivity and apparent chargeability–resistivity can be inverted in a combined manner, using SIS for a common uniform thickness layer earth model to estimate the respective subsurface distributions of resistivity and chargeability–resistivity. The quotient of the two profiles will give the sought after chargeability profile. A brief outline of SIS is provided for completeness. Three theoretical models are included to confirm the efficacy of SIS software by inverting only the synthetic resistivity sounding data. Then one synthetic data set based on a geological model and three field data sets (combination of resistivity and IP soundings) from diverse geological and geographical regions are included as validation of the proposal. It is hoped that the proposed scheme would complement the resistivity interpretation with special reference to shaly sand formations.

1. Introduction

Resistivity sounding data is backbone of groundwater exploration. However, in case of shaly aquifers, the quantitative interpretation of resistivity data occasionally lacks in confidence level. It is not possible to identify the aquifer's shalyness uniquely from the resistivity data. Fortunately, the Induced Polarization (IP) method, primarily based on membrane polarization in non-metallic environments, can be used to provide a possible estimator of shalyness. Although the Direct Current (DC) resistivity and the IP methods of geophysical prospecting were introduced almost at the same time (Schlumberger, 1920), yet the two are currently miles apart in application and reliability of the interpretation of data. Due to the physical understanding and mathematical developments of resistivity prospecting method, it has virtually monopolized the field of groundwater exploration, mineral exploration and civil engineering applications. This has become possible due to the sustained efforts of researchers starting from Stefanesco et al. (1930) (e.g. Ghosh, 1971; Koefoed, 1979; Kunetz, 1966; Langer, 1933; Pekeris, 1940). Although IP also received widespread attention of geophysicists (Dakhnov, 1962; Loeb, 1976; Marshall and Madden, 1959; Vacquier et al., 1957) aiming at strengthening the physical understanding of the method yet, it could not get strength as a viable method except in the case of sulfide minerals. However, it is to be noted that for shaly sand formations IP has great potential for complementary/supplementary information in association with resistivity method for interpretation of subsurface configuration (Börner et al., 1996; Lima and Sri Niwas, 2000; Worthington and Collar, 1984).

Quantitative estimation of parameters of a layered earth model from observed resistivity and/or IP sounding data constitutes a nonlinear inverse problem. The inversion of geoelectrical data is rooted in the pioneer contribution by Langer (1933) who showed that if the ground resistivity continuously varies with depth and the potential distribution about a current electrode grounded at the surface is completely known, then the inverse potential problem has a unique solution. However, a continuous model may be obtained using a continuous inversion technique that requires (i) a Hilbert space for its description, and (ii) operator and function for any operation (Porsani et al., 2001). To circumvent this rigorous exercise, at the cost of uniqueness, a general discrete quasi-linear iterative inversion method is used within a less restrictive framework for obtaining the discrete earth model when the solution to the forward problem is known and when a reasonable initial guess of the model parameter is possible. The model parameters are perturbed from the initial guess and the corresponding change in observables is calculated. As long as the perturbation is small it is expected that the relation between the change in observable and parameter perturbation is linear. The quality of such inversion depends both on the choice of the discretized forward model and on the choice of the optimization algorithm. A

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different forward model and/or a different optimization algorithm will generate a different inversion (Treitel, 1989). Intimately connected to the choice of an optimization algorithm is the choice of the error criterion i.e. how should the difference between what is observed and what is modeled be minimized.

For more than a decade several commercial and non-commercial 1D inversion programs are developed for both time- and frequency-domain IP data with varying inversion strategies (e.g. Loke, 1999; Loke and Barker, 1996; Oldenberg and Li, 1994; Roy, 1999, 2000 etc.). Most of the IP (time-domain) programs use a linear filter based forward modeling. In this approach the data set is generally not used exactly as observed, instead a sampled data set based on sampling interval of the filter coefficients is inverted that implicitly means a priori smoothing of the data. Inversion is accomplished by first quasi-linearizing the nonlinear inverse problem and then using linear inversion technique iteratively having least squares minimization as error criterion, a strategy similar to that of resistivity inversion. This strategy of inverting data sets, usually of inadequate quality and quantity (ill-posed inverse problem), is beset by the endemic problems of non-uniqueness, stability and resolution common to all ill-posed problems (Backus and Gilbert, 1970; Berdichevsky and Zhdanov, 1984). The conventional quasi-linear inversion requires an educated guess of the electrical parameter variation fairly close to the true model, being a serious disadvantage. In the absence of any a priori knowledge, a liberally over parameterized initial guess model just did not succeed (Inman et al., 1973; Wu, 1968) and a thin layer problem had little chance of being resolved adequately (problem of equivalence and suppression; Maillet, 1947).

We are proposing in this paper an extended application of the Straightforward Inversion Scheme (SIS) (Gupta et al., 1997) and Modified Straightforward Inversion Scheme (Sri Niwas et al., 2007) developed for the inversion of Vertical Electrical Sounding data and Magnetotelluric Sounding data respectively, for implicit inversion of IP (time domain) sounding data in combination with Vertical Electrical Sounding data. The mathematical formulation of SIS is completely different from the existing techniques and is based on power series representation of a rational function like resistivity kernel function that reduces the nonlinear inverse problem to a linear one using the model discretization criterion of Kunetz (1972). The linear inverse problem is solved as an underdetermined class using regularized minimum norm optimization algorithm. The inverse solution gives estimates of the coefficients of the apparent resistivity power series to be used in a recurrence relation, to obtain the subsurface resistivity distribution. The method does not require an initial guess and can handle any number of layers as such over-parameterization is no longer a serious problem. Since geometrical parameters (layer thicknesses) are kept out of inversion process the effect of equivalence is minimized (Sri Niwas et al., 2007). Besides these advantages SIS provides nearly continuous resistivity distribution even when using discrete inverse theory. Instead of initial guess the SIS assumes a model space (that remains same for any 1D model) consisting of layers of uniform thickness that must be judiciously chosen keeping in mind the resolution desired. The scheme does not break down in the presence of large errors in data (≈20%) and provides an approximate solution even when data are not consistent with a truly one-dimensional model. For extending the applicability of SIS to IP (time-domain) data, we are taking the help of Seigel's (1959) paper.

Seigel (1959) presented a mathematical formulation that represents the IP sources as dynamic dipoles. He started with the fundamental approach of a volume distribution that directly leads to the concept of change of electrical potential with time, due to either the "normal" induced effect ("normal" or background effect being present in all rocks, consolidated or otherwise, even when totally lacking in metallic sulfide minerals. It lies within relatively narrow limits for most rocks and could, therefore, be resolved from overvoltage effects. Vacquier et al. (1957) added to it the electro-dialysis of clay across semi-permeable partitions formed by sand grains in unconsolidated sediments) or the effect of dissemination of sulfide particles. It was shown that the secondary response due to polarization phenomenon can be represented as that due to volume density of dipolar sources. The current from these sources must obey the normal steady current flow boundary conditions at discontinuities in conductivity. The net effect of IP phenomenon was shown to result in reducing the conductivity,σ, of the medium by a factor $(1-m)$, m being chargeability of the medium. Seigel (1959) concluded, "thus whenever we can obtain the steady state current flow solution for any problem involving conductors whose conductivities σ_i are known, we have also obtained the solution for the same potential problem taking into account the secondary dipolar distributions. The one step required is to change σ_i to $\sigma_i\,(1-m_i)$ in each instance. The difference between the two potential functions will then give us the peak secondary voltages to be experienced from polarization effects". This was a landmark conclusion and based on it, Seigel (1959) suggested a method to compute type curves for interpretation of apparent chargeability data. However, the method as such was very cumbersome, involving computation of derivatives of DC apparent resistivity function with respect to true resistivity of various formations. Patella (1972a, 1972b, 1973), introduced a new parameter, "fictitious resistivity" for the interpretation of time domain induced polarization sounding data. Patella provided encouraging impetus to quantitative interpretation of IP data (Lima and Sri Niwas, 2000; Sastry and Tesfakiros, 2006; Turgay, 1976). However, it is shown in the present paper that every aspect introduced by Patella can also be derived from the dynamic dipolar representation conceived by Seigel (1959). Hence, besides extending the scope of application of SIS (Gupta et al., 1997) to IP data inversion, giving due credit to Seigel, is one of the objectives of our paper. Theoretical developments based on Seigel (1959) in association with the modified SIS (Sri Niwas et al., 2007) can effectively solve the problem of inversion of IP sounding data. Using one synthetic data based on a geologic model and three field data sets, it is shown that combined inversion of DC resistivity and IP sounding data can be easily and efficiently performed to improve the confidence level in case of shaly sand formations.

2. Theoretical considerations

In a homogeneous earth of resistivity ρ_1 and chargeability m_1 if a current I is introduced through a point source placed on the surface of the earth, the potentials U (in the absence of dipolar sources) and U' (in the presence of dipolar sources) at a distance r satisfy the Laplace equation $\nabla^2 U=0$ and $\nabla^2 U'=0$ whose solutions in spherical co-ordinate system can be written as (Keller and Frischknecht, 1966)

$$U = \frac{I\rho_1}{2\pi}\frac{1}{r} \tag{1}$$

and (Seigel, 1959)

$$U' = \frac{I\rho_1}{2\pi(1-m_1)}\frac{1}{r} \approx \frac{I\rho_1(1+m_1)}{2\pi}\frac{1}{r} \quad (m_1 \ll 1 \text{ in case of "normal" induced effect}) \tag{2}$$

respectively.

The excess polarization potential $Up=U'-U$, can be obtained as

$$U_p = \frac{I}{2\pi}m_1\rho_1\frac{1}{r}. \tag{3}$$

This confirms that polarization potential U_p also satisfies the Laplace equation $\nabla^2 U_p=0$. Analogous to the resistivity during current on situation, for the homogeneous half space we define a parameter $\rho_p=m_1\rho_1$ termed as 'chargeability–resistivity' that can be conceptually