Journal of Applied Geophysics 78 (2012) 31–43

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Taming the non-linearity problem in GPR full-waveform inversion for high contrast media ☆

Giovanni Meles [a,*], Stewart Greenhalgh [a,b], Jan van der Kruk [c], Alan Green [a], Hansruedi Maurer [a]

[a] Institute of Geophysics, ETH Zurich, Sonneggstrasse 5, 8092, Zurich, Switzerland
[b] Department of Physics, University of Adelaide, 5005 Adelaide, South Australia
[c] Forschungszentrum Julich, 52425 Julich, Germany

ARTICLE INFO

Article history:
Received 18 October 2010
Accepted 3 January 2011
Available online 26 December 2011

Keywords:
GPR
FDTD
Inversion
Time-domain
Frequency-domain
Bandwidth expansion
Stability

ABSTRACT

We present a new algorithm for the inversion of full-waveform ground-penetrating radar (GPR) data. It is designed to tame the non-linearity issue that afflicts inverse scattering problems, especially in high contrast media. We first investigate the limitations of current full-waveform time-domain inversion schemes for GPR data and then introduce a much-improved approach based on a combined frequency-time-domain analysis. We show by means of several synthetic tests and theoretical considerations that local minima trapping (common in full bandwidth time-domain inversion) can be avoided by starting the inversion with only the low frequency content of the data. Resolution associated with the high frequencies can then be achieved by progressively expanding to wider bandwidths as the iterations proceed. Although based on a frequency analysis of the data, the new method is entirely implemented by means of a time-domain forward solver, thus combining the benefits of both frequency-domain (low frequency inversion conveys stability and avoids convergence to a local minimum; whereas high frequency inversion conveys resolution) and time-domain methods (simplicity of interpretation and recognition of events; ready availability of FDTD simulation tools).

1. Introduction

Ground-penetrating radar (GPR) finds wide application in diverse areas of civil engineering and environmental investigations, such as buried utilities mapping, concrete and pavement inspection, rail track surveillance, UXO detection, hydrology, sedimentology, etc. The technique is also popular in archaeology and glaciology, as witnessed by the large number of such papers recently presented at the 13th International GPR conference in Lecce, Italy (GPR, 2010). Surface implementations of the technique largely rely on migration algorithms (Heinke et al., 2005; Streich et al., 2006; van der Kruk et al., 2003) to image the geometry of buried targets from the scattered signals. Such reflector detection and delineation schemes are akin to wavefield migration procedures commonly used in the more mature field of seismic exploration (Claerbout, 1985; Yilmaz and Doherty, 2001) and to focussed-lag sum processors used in the early days of microwave medical imaging (Fear and Stuchly, 2000; Hagness et al., 1998). These migration-style schemes use the full waveforms, but they stop short of an actual inversion in that they do not fully recover the medium (electrical) properties. By contrast, crosshole GPR studies have been mainly based on first arrival traveltime and amplitude tomography using the direct transmitted arrivals to image the relative permittivity ε_r and conductivity σ variations in the interhole medium (e.g., Carlsten et al., 1995; Clement and Barrash, 2006; Fullagar et al., 2000; Musil et al., 2006; Olsson et al., 1992; Tronicke et al., 2001). Because such image reconstruction procedures use only a small amount of the available information, they provide only limited resolution. Imaging low velocity (high permittivity) zones is especially difficult because first arrival raypaths tend to by-pass such features. Full-waveform inversion offers the promise of far better imaging capabilities. Early versions of full-waveform electromagnetic (EM) inversion (both radar and microwave) were based on the Born approximation of weak scattering (i.e., for low contrast targets), thus neglecting secondary interactions between obstacles (Chew and Wang, 1990; Wang and Chew, 1989). This linearised the problem. Furthermore, it was often assumed that the background medium was homogeneous, for which analytic Green's functions were available. Similar assumptions were incorporated in early seismic inversion approaches. The pioneering seismic waveform papers by Tarantola (1986) and Mora (1987) did not impose such restrictions. These fully elastodynamic seismic inversion schemes suffered from limited computational resources available at the time, and were not adopted until 10–20 years later (Charara et al., 1996, 2000; Plessix, 2008).

Kuroda et al. (2007) and Ernst et al. (2007a) were among the first researchers to tackle theoretically, crosshole full-waveform GPR

☆ A publishers' error resulted in this article appearing in the wrong issue. The article is reprinted here for the reader's convenience and for the continuity of the special issue. For citation purposes, please use the original publication details: Meles, G., et al., Taming the non-linearity problem in GPR full-waveform inversion for high contrast media, J. Appl.Geophys. (2011), doi:10.1016/j.jappgeo.2011.01.001.

* Corresponding author.
E-mail address: meles@aug.ig.erdw.ethz.ch (G. Meles).

0926-9851/$ – see front matter © 2011 Published by Elsevier B.V.
doi:10.1016/j.jappgeo.2011.12.001

inversion as a non-linear iterative problem, albeit in two dimensions. Kuroda et al. (2007) inverted only for the relative permittivity ε_r. Ernst et al. (2007a) used a stepped (cascaded) inversion scheme, whereby the ε_r distribution was first updated while the σ distribution was held fixed, and then the σ values were updated holding the ε_r values fixed. Both approaches used a finite-difference time-domain solution of Maxwell's equations and a gradient-based algorithm. Rather than calculating the sensitivities explicitly, as in a Gauss–Newton inversion approach, they used the zero-lag cross-correlation between the forward propagated field and the back-propagated residual field at the receivers to calculate the gradient directions. Ernst et al. (2007b) successfully applied the technique to observed data from two field sites.

Earlier and parallel developments occurred in biomedical microwave tomography in both the time domain and the frequency domain, using iterative and distorted Born approaches (Wang and Chew, 1989), as well as other more refined procedures (Fhager et al., 2005; Fhager and Persson, 2005; Gustafson and He, 2000; Hashemzadeh et al., 2006; Rubaek et al., 2007; Tanaka et al., 1999). State-of-the-art microwave imaging is described by Dubois et al. (2009), Rubaek et al. (2009) and Solvodieri (2010). It should be appreciated that in the microwave case, the target lies in either air or de-ionised water (i.e., homogeneous media) and is completely surrounded (360°) by the antennas. This is almost never the case in GPR, where the host material is heterogeneous and the angular coverage is limited. Furthermore, it is assumed in microwave imaging that the transmitters and receivers are polarised in the 2D medium–invariant transverse or y-direction, such that the EM equations (transverse electric or TE case) for a line source simplify considerably to scalar wave equations involving a single E-field component (E_y). This is only applicable in GPR for surface recording in which the antennae are directed perpendicular (y direction) to the profile (x) direction and the geology is two dimensional. For antennae oriented in the saggital plane or for 3D media, such equations cannot be used, thus seriously limiting such approach for more general GPR applications. Some of the microwave imaging algorithms developed in recent times (e.g., Dubois et al., 2009) make the further assumption that the scattered field can be isolated from the total field. As a consequence, the incident field is known because measurements can be made with and without the target (object) present (Fhager et al., 2005). Separating the direct wave from the scattered field is sometimes possible with careful time gating of surface GPR data, but it is extremely problematic with crosshole data, in which the various arrivals overlap. Moreover, the subsurface targets cannot be "removed" in earth science applications.

The transverse magnetic (TM) case with the electric field polarised in the x–z plane of propagation and the magnetic field in the transverse or y-direction, requires a full-vector treatment (E_x and E_z components). This was recently given by Meles et al. (2010), thus enabling for the first time, the joint inversion of surface data (antennae oriented in the x direction) and crosshole data (antennae oriented in the z direction). These authors also described a new scheme that simultaneously updates ε_r and σ estimates, leading to improved performance and efficiency over the cascaded scheme of Ernst et al. (2007a, b). Although the Ernst et al. (2007a, b) and Meles et al. (2010) schemes both offer sub-wavelength resolution when the target coverage is favourable, it should be appreciated that the full-waveform GPR inverse problem is both ill-posed and non-linear. Notwithstanding the sophistication of the new schemes, the non-linearity of the forward problem can cause them to fail to provide a satisfactory picture of the subsurface. It is well known that the non-linearity is mainly associated with multiple scattering (Mora, 1987), being particularly severe when the differences between the true model and the current (starting or guessed) model are large in terms of the target contrasts (ε_r and σ) and target size. Large anomalous bodies having appreciable velocity contrasts with their surroundings cause significant traveltime differences between the observed traces and those computed for the background model. When the time shifts exceed more than half a period, the inversion can get trapped in local minima.

One solution to the local minimum problem is the frequency hopping method used in microwave imaging (Chew and Lin, 1995; Dubois et al., 2009). The inversion starts at a low frequency and progressively moves to a higher frequency, using the model from the previous frequency inversion as the starting model for the next higher frequency. A similar approach has been proposed for frequency-domain seismic inversion (Maurer et al., 2009; Pratt et al., 1998; Zhou and Greenhalgh, 2003), in which inversions are carried out one frequency at a time. In realistic situations, the low frequency data may be contaminated by noise, such that the frequency hopping approach is unstable. Until now, the alternative was to work in the time domain with wide-band transient signals and to impose prior constraints to the model space while retaining the full bandwidth of the data. Imposing smoothness constraints on the model space addresses the second issue of ill-posedness of the geophysical inverse problem, but it does not resolve the non-linearity problem; it simply stabilises or regularises the problem. To mitigate the non-linearity issue, one may invoke à priori information on the ε_r and σ distributions so that the initial model is close to reality. This can work well in biomedical applications (Fhager and Persson, 2007), in which reasonable knowledge exists on the shape, location, and likely contrasts of the targets (e.g., organs) and surrounding structures. Unfortunately, for most GPR applications, such information is not available. One approach (Ernst et al., 2007a) that attempts to take into account prior information is to use the results of traveltime and amplitude tomography (e.g., Fullagar et al., 2000; Musil et al., 2006) as the starting model. This helps in some cases, but our numerous synthetic experiments demonstrate that such an approach yields unsatisfactory results for complex models due to the inherent limitations of ray-based methods (maximum achievable resolution, difficulties in mapping large velocity contrast inclusions and certain types of low velocity structure). An alternative approach is therefore required.

We present here a new full-waveform inversion scheme for GPR data that is based on a combined frequency-time-domain approach. It requires no specific assumptions about the model to be used. The essence of the method is to progressively expand the bandwidth of the data as iterations proceed, starting with low frequencies and successively adding higher frequencies. Only in the final stages of the inversion is the full bandwidth of the data utilised. Although the applications presented here are for 2D models and combined surface-crosshole configurations, the method is theoretically valid for 2D and 3D problems and data collected in any source–receiver configuration. The current application to only 2D problems is imposed by the excessive CPU/memory costs for 3D simulations (i.e., not by any theoretical assumption in the derivation of the forward/inversion scheme). The results of the synthetic examples presented in this paper clearly show that the new method can provide detailed and reliable images of the investigated media, even for high contrast and complex-shaped inclusions.

2. Inversion of GPR data

The minimum set of electrical parameters required to characterise the subsurface completely comprises the permittivity ε_r, conductivity σ and permeability μ distributions. Throughout this paper, permeability is assumed to be constant and equal to the free space value μ_0. Least-squares full-waveform tomographic inversion schemes involve finding the spatial distributions of ε_r and σ that minimise a cost function that is the squared norm of the misfit between the simulated and the observed GPR traces. In the next section, we briefly introduce a recently developed full-waveform time-domain inversion scheme and discuss its limitations in terms of spectral coverage and stability. Subsequently, we present a very simple yet illustrative example of