Journal of Applied Geophysics 78 (2012) 52–67

Contents lists available at SciVerse ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

2-D joint structural inversion of cross-hole electrical resistance and ground penetrating radar data

Abderrezak Bouchedda [a,*], Michel Chouteau [a], Andrew Binley [b], Bernard Giroux [c]

[a] Departement CG&M, Ecole Polytechnique, CP 6079 succ. Centre-Ville, Montreal, Quebec, Canada H3C 3A7
[b] Lancaster Environment Centre, Lancaster University, Lancaster, LA1 4YQ, United Kingdom
[c] Institut National de Recherche Scientifique, Centre Eau Terre Environnement, 490, de la Couronne, Québec, Québec, Canada G1K 9A9

ARTICLE INFO

Article history:
Received 11 December 2010
Accepted 14 October 2011
Available online 21 October 2011

Keywords:
ERT
RTT
Joint structural inversion
Wavelet thresholding

ABSTRACT

We present a joint structural inversion algorithm for cross-hole electrical resistance tomography (ERT) and cross-hole radar travel time tomography (RTT) that encourages coincident sharp changes on a smoothly varying background in the two models. The proposed approach is based on the combination of two iterative soft-thresholding inversion algorithms in parallel manner where the structural information is exchanged at each iteration. Iterative thresholding algorithm allows to obtain a sparse wavelet representation of the model (blocky model) by applying a thresholding operator to the wavelet coefficients of model obtained through a Gauss–Newton iteration. The structural information is introduced in the inversion system using the smoothness weighting matrices that control boundary cells and the thresholds that are estimated by maximizing a structural similarity criterion, which is a function of the two (ERT and RTT) models. A Canny edge detector is implemented to extract the structural information. The detected edges serve to build a weighting matrix that is used to alter the smoothness matrix constraint. To validate our methodology and its implementation, tests were performed on three synthetic models. The results show that the parameters estimated by our joint inversion approach are more consistent than those from individual inversions and another joint inversion algorithm. In addition, our approach appears to be robust in high noise level conditions. Finally, the proposed algorithm was applied for vadose zone characterisation in a sandstone aquifer. It achieves results that are consistent with hydrogeological information and geophysical logs available at the site. The results were also compared in terms of structural similarities to models obtained by a joint structural inversion algorithm with a cross-gradient constraint. Based on this comparison and hydrogeologic information, we conclude that the proposed algorithm allows to the RTT and ERT models to be dissimilar in the areas where the data are incompatible.

1. Introduction

Cross-hole electrical resistance tomography (ERT) and cross-hole ground penetrating radar travel time tomography (RTT) have become increasingly used in hydrogeological, environmental and geotechnical applications (Butler, 2005; Rubin and Hubbard, 2005; Vereecken et al., 2006). Electrical properties estimated by these techniques (electrical conductivity and dielectric permittivity, respectively) are linked to hydraulic parameters such as porosity, water content, salinity, and cation exchange capacity (e.g. Lesmes and Friedman, 2005). To build a better understanding of the hydrogeological setting, it is important to have the most accurate geophysical characterization; when compared to the geophysical understanding obtained by individual inversion of both ERT and RTT datasets improvements to the estimation of the final model can be achieved by appropriate regularization schemes, *a priori* information and use of joint inversion.

The term joint inversion has been used in several ways in geophysical literature. In the present study, we define joint inversion as estimation of subsurface model expressing different related geophysical properties using several independent geophysical data types. The geophysical properties can be linked by petrophysical relationship (Lines et al., 1988) or structural similarities (e.g. Haber and Oldenburg, 1997) or statistical relationship (e.g. Bosch, 1999). We call it *simultaneous joint inversion* when all data types are inverted using a single objective function and *cooperative joint inversion* when the separate data inversions are linked via *a priori* or *a posteriori* information. The cooperative formulation suffers mainly from the bias that can be introduced by bad *a priori* information and the simultaneous formulation suffers mainly from the weighting of the individual data (Lines et al., 1988).

Joint inversion of geophysical data has been reported in many papers (Bosch, 1999; Doetsch et al., 2010; Gallardo and Meju, 2003; Gallardo et al., 2005; Günther and Rücker, 2006; Haber and Oldenburg, 1997; Kozlovskaya, 2000; Lines et al., 1988; Moorkamp

* Corresponding author. Tel.: +1 514 340 4711x5233; fax: +1 514 340 3970.
E-mail addresses: bouchedda@geo.polymtl.ca (A. Bouchedda), chouteau@geo.polymtl.ca (M. Chouteau), a.binley@lancaster.ac.uk (A. Binley), bernard.giroux@ete.inrs.ca (B. Giroux).

0926-9851/$ – see front matter
doi:10.1016/j.jappgeo.2011.10.009

et al., 2011; Paasche and Tronicke, 2007; Sasaki, 1989; Vozoff and Jupp, 1975). When the relationship between physical properties is complex, unknown, spatially variable or very weak, joint inversion might be possible using structural constraints (Gallardo and Meju, 2003; Günther and Rücker, 2006; Haber and Oldenburg, 1997), zonation (Hyndman et al., 1994; Paasche and Tronicke, 2007) or stochastic approaches (Bosch, 1999). Note that the structural information is related to assumptions of coherent spatial variations of physical properties.

Both resistivity and relative permittivity will be dependent on moisture content and mineralogy (Lesmes and Friedman, 2005), although the relationship between the two can be complex. Based on petrophysical models, Linde et al. (2006) demonstrated that under some conditions the variation of electrical conductivity and relative permittivity is affected by variations of the same controlling rock properties factors. Hence, the structural similarity between the two geophysical models can be either complete or partial (only in some places). In addition, it has been shown (Day-Lewis et al., 2005) that these models have complementary resolution. The region of highest resolution is localized close to boreholes for ERT, and in the central part of the investigated area for RTT. To benefit from the aforementioned complementarities, Linde et al. (2006) apply a structural constraint that consists in imposing the cross product of the gradients of the two models to be zero. The cross-gradient joint inversion approach has been originally proposed by Gallardo and Meju (2003) to invert ERT and seismic refraction data and has been successfully applied to other geophysical inverse problems (e.g. Gallardo et al., 2005; Gallardo and Meju, 2007; Linde et al., 2006, 2008; Moorkamp et al., 2011; Tryggvason and Linde, 2006). Günther and Rücker (2006) proposed a combined separate inversion where the combination of both models is accomplished by mutually controlling the structural weights based on the principles of robust modeling. Lelievre (2009) proposed a very similar approach by imposing the same structural weight constructed from the sum of the gradients of the two models. In the present contribution, we work in the wavelet domain, where sharp discontinuities are encouraged. In this way, the structural information may be easier to extract from the inverted models. In order to achieve this goal, we use an edge detection technique based on image gradient. Note that we assume that a model with relatively homogenous zones (blocky model) can describe the subsurface.

The paper is divided into three main sections. In the first part we provide a brief review of the theory for ERT and RTT modeling and inversion. The regularized ERT inverse problem is presented as the minimization of the Tikhonov parametric functional with a Gauss–Newton algorithm. In the second section of the paper we give a brief description of the developed algorithm. Finally, in the third part, the new algorithm is applied to synthetic and field data to assess its reliability and performance.

2. Inversion method

In this section, we develop the inversion methodology and algorithms for carrying out the soft thresholding iterative reconstruction of Eso et al. (2008). Our inversion is carried out in 2-D. The resistivity and slowness models consist of cells where the physical property is constant within each cell.

Both ERT and RTT inverse problem are nonlinear and ill posed. The mathematical formalism of the regularized linearized ERT and RTT inverse problem is a minimization of the Tikhonov parametric functional, consisting of a data misfit term and a regularization term which is generally the L_2-norm of the spatial changes in the model parameters (e.g. Zhdanov, 2002). In our case, we are looking for blocky structures that are hard to obtain when using the minimization of the L_2-norm of the spatial changes in the model parameters. An alternate method is to use a general measure, like the L_1-norm that tends to produce models consisting of areas with piecewise constant model values. One way would be to use robust estimators as proposed by Claerbout and Muir (1973). The inverse problem can now be solved using iterative reweighted least-squares techniques applied to the minimization of the following weighted model functional

$$\varphi(\mathbf{m}) = ||\mathbf{W_d}(\mathbf{d} - \mathrm{F}(\mathbf{m}))||_2^2 + \beta||\mathbf{W_c}\mathbf{C}\cdot(\mathbf{m} - \mathbf{m}_{\mathrm{ref}})||^2, \tag{1}$$

where $\mathbf{m}$ is the model parameters and $\mathbf{d}$ is the observed data. For ERT inversion $\mathbf{m}$ and $\mathbf{d}$ correspond to the resistivity ρ and apparent resistivity or resistance, respectively. For RRT inversion, $\mathbf{m}$ and $\mathbf{d}$ correspond to slowness s and travel time t, respectively.

The first term in Eq. (1) is the misfit functional, which is a measure of misfit between the theoretical values $\mathrm{F}(\mathbf{m})$ and the observed data $\mathbf{d}$. The second term is the model objective function or stabilizing functional, which quantifies desirable features of the resulting distribution of model parameters, and β determines the relative importance of the data misfit and the model objective function. $\mathbf{W_c}$ is the weighting diagonal matrix that represents penalty factors for the different model cell boundaries. The regularization matrix $\mathbf{C}$ is defined as the combination of the identity matrix and the matrix of the first or second derivative of the parameter. It can be written as

$$\mathbf{C} = \alpha_x \mathbf{W}_x \mathbf{D}_x + \alpha_z \mathbf{W}_z \mathbf{D}_z + \alpha_s \mathbf{I} \tag{2}$$

where :

$\mathbf{D_x}$	first or second derivative matrix in *x*-direction
$\mathbf{D_z}$	first or second derivative matrix in *z*-direction
$\mathbf{W_x}$	structural weighting matrix in *x*-direction
$\mathbf{W_z}$	structural weighting matrix in *z*-direction
$\mathbf{I}$	identity matrix
α_x, α_z	smoothing weight factor in *x*- and *z*-direction, respectively
α_s	smallness or closeness weight factor.

Spatially flat or smooth models result from the application of the first derivative or second derivative regularization matrix, respectively. If there is no reference model ($\mathbf{m_{ref}} = 0$), the identity matrix attempts to force the inversion to recover the smallest model, that is model with low model parameters values. When a reference model is incorporated in the Eq. (2), this term ensures that the final model exhibits a small departure from reference model $\mathbf{m_{ref}}$. Note that to produce blocky models, m_{ref} should be homogeneous or blocky. The reduction of the value of α_x or α_z will result in models that are preferentially smooth in *z*- or *x*-direction respectively. For example, a layered model can be obtained using $\alpha_x = 1$ and $\alpha_z = 0.01$. The structural weighting matrices ($\mathbf{W}_x$, $\mathbf{W}_z$) are diagonal matrices where the individual values represent a penalty factors for the corresponding cell boundaries. They can be used to introduce any structural constraint, such as interfaces and edges, in the resulting distribution of model parameters. It contains small weights at the position of edges, and weights equal to one otherwise.

The minimization of the objective functions (1) using a Gauss–Newton algorithm results in the following iterative equations (e.g. Farquharson and Oldenburg, 1998):

$$\Delta\mathbf{m}_i = \left(\mathbf{J}^T\mathbf{W}_d^T\mathbf{W}_d\mathbf{J} + \beta\cdot\mathbf{C}^T\mathbf{W}_c^T\mathbf{W}_c\mathbf{C}\right)^{-1}\left(\mathbf{J}^T\mathbf{W}_d^T\mathbf{W}_d(\mathbf{d} - F(\mathbf{m}_i)) - \beta\cdot\mathbf{C}^T\mathbf{W}_c^T\mathbf{W}\mathbf{C}(\mathbf{m}_i - \mathbf{m}_{\mathrm{ref}})\right) \tag{3}$$

where $\mathbf{J}$ is the Jacobian matrix and $\mathbf{J}^\mathbf{T}$ is the transpose of matrix $\mathbf{J}$.

In the case of ERT, the Jacobian matrix is computed using differential calculus and the Green's functions of the 2.5D Helmholtz equation are obtained according to Zhou and Greenhalgh (1999). Traveltime radar data are inverted using a ray-based approach. The curved ray tracing routine of bh_tomo (Giroux et al., 2007) is used for the forward modeling of radar travel time data and the calculation of the Jacobian matrix. It is based on Huygens principle and on graph theory