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# Analysis of group-velocity dispersion of high-frequency Rayleigh waves for near-surface applications

Yinhe Luo [a,*], Jianghai Xia [b], Yixian Xu [c], Chong Zeng [b]

[a] Institute of Geophysics and Geomatics, China University of Geosciences, Wuhan, Hubei 430074, China
[b] Kansas Geological Survey, The University of Kansas, 1930 Constant Avenue, Lawrence, KS 66047–3724, USA
[c] State Key Laboratory of Geological Processes and Mineral Resources, Institute of Geophysics and Geomatics, China University of Geosciences, Wuhan, Hubei 430074, China

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ABSTRACT

The Multichannel Analysis of Surface Waves (MASW) method is an efficient tool to obtain the vertical shear (S)-wave velocity profile using the dispersive characteristic of Rayleigh waves. Most MASW researchers mainly apply Rayleigh-wave phase-velocity dispersion for S-wave velocity estimation with a few exceptions applying Rayleigh-wave group-velocity dispersion. Herein, we first compare sensitivities of fundamental surface-wave phase velocities with group velocities with three four-layer models including a low-velocity layer or a high-velocity layer. Then synthetic data are simulated by a finite difference method. Images of group-velocity dispersive energy of the synthetic data are generated using the Multiple Filter Analysis (MFA) method. Finally we invert a high-frequency surface-wave group-velocity dispersion curve of a real-world example. Results demonstrate that (1) the sensitivities of group velocities are higher than those of phase velocities and usable frequency ranges are wider than that of phase velocities, which is very helpful in improving inversion stability because for a stable inversion system, small changes in phase velocities do not result in a large fluctuation in inverted S-wave velocities; (2) group-velocity dispersive energy can be measured using single-trace data if Rayleigh-wave fundamental-mode energy is dominant, which suggests that the number of shots required in data acquisition can be dramatically reduced and the horizontal resolution can be greatly improved using analysis of group-velocity dispersion; and (3) the suspension logging results of the real-world example demonstrate that inversion of group velocities generated by the MFA method can successfully estimate near-surface S-wave velocities.



## 1. Introduction

Near-surface shear (S)-wave velocity structure is important in ground-motion amplification and site response in sedimentary basins (Borcherdt, 1970; Stephenson et al., 2005). The Multichannel Analysis of Surface Waves (MASW) method is an efficient tool to obtain the vertical S-wave profile using the dispersive characteristic of Rayleigh waves (Calderón-Macías and Luke, 2007; Luo et al., 2007, 2008a, 2008b, 2009a, 2009b, 2009c; Song et al., 1989; Xia et al., 1999, 2002a, 2004, 2006a). Recent studies associated with Rayleigh-wave data analysis and applications include: bedrock mapping (Miller et al., 1999); near-surface quality factors ($Q$) (Xia et al., 2002b); inversion with the incorporation of higher mode data (Beaty et al., 2002; Luo et al., 2007; Xia et al., 2003); cavern detection (Xia et al., 2004); numerical modeling (Mittet, 2002; Xu et al., 2007); joint inversion of P-wave velocities and surface wave phase velocities (Dal Moro and Pipan, 2007; Ivanov et al., 2006); discussion on resolution of surface-wave data (Xia et al., 2005); a non-layered-earth model (Gibson half-space, Xia et al., 2006a); the nearest offset and cutoff frequencies and their applications (Xia et al., 2006b; Xu et al., 2006, 2009); discussion of Rayleigh-wave inversion with a high-velocity-layer intrusion model (Calderón-Macías and Luke, 2007); a low-velocity-layer intrusion model (Liang et al., 2008; Lu et al., 2007); dispersive imaging (Luo et al., 2008a; Xia et al., 2007); mode separation (Luo et al., 2009a); and assessment of inverted models (Xia et al., 2010).

All recent research and near-surface applications mainly apply Rayleigh-wave phase-velocity dispersion for S-wave velocity estimation with a few exceptions applying Rayleigh-wave group-velocity dispersion (e.g., Liner et al., 2009; Long, 2001; Pedersen et al., 2003; Tillmann, 2005), while both phase velocity and group velocity received the same interest by crustal and earthquake seismologists for deep earth S-velocity estimation (e.g., Dziewonsky et al., 1969; Hartse et al., 1997; Li et al., 2009; Shapiro et al., 2005). This may be because analyzing surface-wave data mainly focuses on phase-velocity dispersive energy imaging (e.g., Luo et al., 2008a; McMechan and Yedlin, 1981; Park et al., 1998; Xia et al., 2007; Yilmaz, 1987). Recently, several researchers present successful results using Rayleigh-wave group-velocity dispersion for near-surface applications. Long (2001) used surface-wave

* Corresponding author. Tel.: +86 15071243298.
*E-mail addresses:* luoyinhe@cug.edu.cn, luoyinhe@gmail.com (Y. Luo).

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group-velocity tomography to delineate a suspected 4-meter burial trench. Pedersen et al. (2003) tested the robustness of the reassigned Multiple Filter Analysis (MFA, Dziewonsky et al., 1969) to synthetic and real-world surface-wave data to generate group-velocity dispersive energy. Tillmann (2005) presented an unsupervised wavelet transform method for simultaneous inversion of multimode surface-wave group-velocity dispersion curves. Liner et al. (2009) reported an appropriately tuned continuous wavelet transform to image group-velocity curves in shallow water. Landès et al. (2009) investigated the feasibility of ambient-noise surface-wave tomography using marine noise data to extract the information about S-wave structure of the superficial seafloor.

Herein, we first compare sensitivities of fundamental surface-wave phase velocity with group velocity with three four-layer models including a low-velocity layer (LVL) or a high-velocity layer (HVL). Then we use the MFA method to image group-velocity dispersive energy of synthetic data simulated by a finite difference method (Xu et al., 2007). Finally we invert a high-frequency surface-wave group-velocity dispersion curve generated from real-world data.

## 2. Sensitivity analysis

The main task of sensitivity analysis is to define sensitivities of fundamental surface-wave phase- and group-velocities to S-wave velocities at different depths. For a given layered-earth model, the analytical phase-velocity dispersion curves can be calculated by the Knopoff method (Schwab and Knopoff, 1972). The relationship between group velocity and phase velocity is $\mathrm{Vg}(f) = \mathrm{Vr}(f) + \mathrm{k}(f)\frac{\partial \mathrm{Vr}(f)}{\partial \mathrm{k}(f)}$, where $\mathrm{Vr}(f)$, $\mathrm{Vg}(f)$, and $\mathrm{k}(f)$ are the phase velocity, the group velocity, and the wavenumber at frequency $f$, respectively. The analytical group-velocity dispersion curves can be calculated by program *sdisp96* that is part of *Computer Programs in Seismology* (Herrmann and Ammon, 2004).

Sensitivity of phase/group velocity to S-wave velocity changes can be numerically addressed by analyzing the Jacobian matrix of Rayleigh-wave function (Xia et al., 1999, 2003) or by a simple concept of relative difference (Feng et al., 2001). Here we apply the concept of relative difference that is defined as

$$S_{ri} = \frac{1}{\mathrm{Vr}(f)}\frac{\partial \mathrm{Vr}(f)}{\partial \mathrm{V}_{si}}\delta \mathrm{V}_{si} \approx \frac{\mathrm{Vr}(f, \mathrm{V}_{si} + \alpha \cdot \mathrm{V}_{si}) - \mathrm{Vr}(f, \mathrm{V}_{si})}{\mathrm{Vr}(f, \mathrm{V}_{si})} \cdot 100\%, \text{ and} \quad (1)$$

$$S_{gi} = \frac{1}{\mathrm{Vg}(f)}\frac{\partial \mathrm{Vg}(f)}{\partial \mathrm{V}_{si}}\delta \mathrm{V}_{si} \approx \frac{\mathrm{Vg}(f, \mathrm{V}_{si} + \alpha \cdot \mathrm{V}_{si}) - \mathrm{Vg}(f, \mathrm{V}_{si})}{\mathrm{Vg}(f, \mathrm{V}_{si})} \cdot 100\% \quad (2)$$

where $V_{si}$ is the S-wave velocity of the $i$th layer and $\alpha$ is a perturbation factor applied to the model parameters. $S_{ri}$ and $S_{gi}$ are sensitivities of fundamental-mode phase and group velocities to S-wave velocity of the $i$th layer, respectively. In practice, a 10% change of S-wave velocity is considered to analyze sensitivity and $V_P$, thickness, and density are kept unchanged (Feng et al., 2005). Figs. 1–3 show the sensitivity of fundamental-mode phase- and group-velocities to the S-wave velocity of each layer of models in Table 1.

The first model in Table 1 represents a normal layered model with S-wave velocity increasing with depth. Fig. 1a shows that sensitivities of the fundamental-mode phase velocities in the first layer are highest, and usable frequency (high sensitivity) ranges for the layer are widest in four layers. Sensitivities of the fundamental-mode phase velocities in the third layer are the lowest and usable frequency ranges for the layer are the narrowest in four layers. Sensitivity of group velocity (Fig. 1b) shows two main differences from that of phase velocity (Fig. 1a): (1) sensitivities of the fundamental-mode group velocities are higher than those of the fundamental-mode phase velocities in all layers. The highest sensitivity in Fig. 1a is about 12% while in Fig. 1b is over 15%, which exists in the first three layers; and (2) usable frequency ranges of the fundamental-mode group velocities are wider than those of the fundamental-mode phase velocities in all layers.

The second model in Table 1 contains a LVL (the second layer). Fig. 2a shows that sensitivities of the fundamental-mode phase velocities in the LVL are highest, and usable frequency ranges for the LVL are widest in four layers. Sensitivities of the fundamental-mode phase velocities in the layer below the LVL are the lowest and usable frequency ranges for that layer are the narrowest in four layers. Sensitivity of group velocity (Fig. 2b) shows two main differences from that of phase velocity (Fig. 2a): (1) sensitivities of the fundamental-mode group velocities are higher than those of the fundamental-mode phase velocities in all layers especially in the third layer (below the LVL); (2) usable frequency ranges of the fundamental-mode group velocities are wider range than those of the fundamental-mode phase velocities in all layers. Liang et al. (2008) analyzed inversion stability of multimode Rayleigh-wave dispersion curves using a LVL model and concluded that, because a LVL traps most Rayleigh-wave energy, Rayleigh-wave phase velocities are insensitive to variations in the layer below the LVL.

The third model in Table 1 contains a HVL (the second layer). Sensitivities of the fundamental-mode group velocities (Fig. 3b) are higher than those of the fundamental-mode phase velocities (Fig. 3a) and usable frequency ranges of the fundamental-mode group velocities are wider than those of the fundamental-mode phase velocities in all layers. Unlike Fig. 2, sensitivities of the fundamental-mode phase/group velocities in the layer below the HVL (Fig. 3) are not the lowest, and usable frequency ranges for that layer are not the narrowest in four layers, which is very helpful for S-wave velocity estimation. Inversion of phase velocities, however, is still difficult when a HVL presents in a layered model. Calderón-Macías and Luke (2007) pointed out that inversion of phase-velocity measurements of Rayleigh-wave energy for sites containing stiff layers can be erroneous if such layers are not characterized in the starting or reference model.

In general, sensitivities of group velocities are higher than those of phase velocities and usable frequency ranges for group velocities are

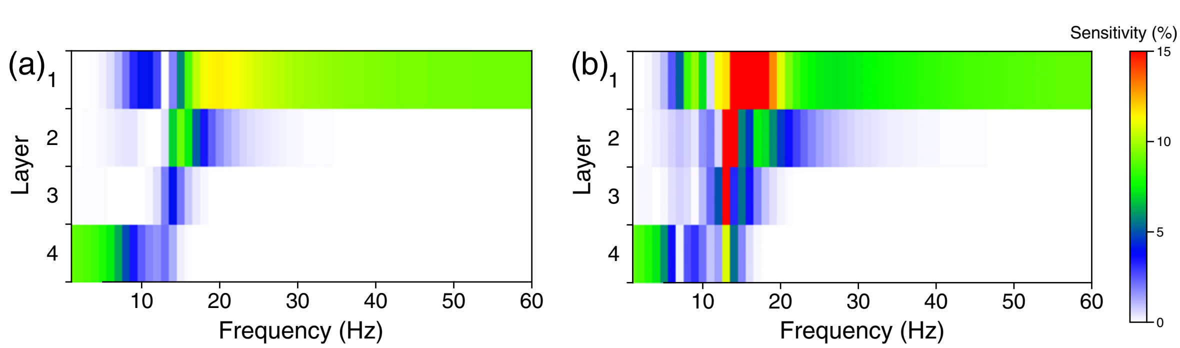


**Fig. 1.** (a) Phase-velocity and (b) group-velocity sensitivity of the fundamental mode for mode 1 (Table 1), S-wave velocity increasing with depth.