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# A mixture-theory-based dynamic model for a porous medium saturated by two immiscible fluids

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**Abstract**

In terms of the mixture theory, a linear isothermal dynamic model for a porous medium saturated by two immiscible fluids is developed in the paper. The volume fraction of each phase is characterized by the saturation of the wetting phase and the porosity of the porous medium. The mass and momentum balance equations are obtained according to the generalized mixture theory. The isothermal constitutive relations for the stress, the pore pressure are derived from the entropy inequality of the porous medium. Three kinds of attenuation mechanisms are introduced in terms of the entropy inequality. The drag force model is introduced to account for the attenuation due to global fluid flow between the fluids and the solid skeleton, while the capillary pressure relaxation and the porosity relaxation mechanism are used to describe the relaxation process related to the variation of the saturation and the porosity. The capillary pressure relaxation mechanism is related to relaxation of the interface between the two fluids, while the porosity relaxation mechanism is related to the local fluid flow of the porous medium. In terms of the proposed model, a linear model for the two-fluid saturated porous medium is presented in the paper. The physical meaning and the evaluation of the material parameters in the linear model are discussed in the paper. In terms of the established two-fluid linear model, the velocities and the attenuations for different wave modes are calculated. It follows from numerical results that the capillary pressure relaxation and the porosity relaxation have a significant influence on the attenuations of the P wave modes.


*Keywords:* Porous media; Immiscible fluids; Dynamic model; Capillary pressure; Entropy inequality

## 1. Introduction

Based on the work of von Terzaghi (1923), Biot (1956a,b, 1962) presented a theoretical description of a porous medium saturated by one fluid. In deriving the equations of motion for the porous medium, Biot introduced the Lagrangian viewpoint and the concept of generalized coordinates. Biot derived the constitutive relation of the porous medium from a single free energy. Biot also extended his theory to the anisotropic medium (Biot, 1955), poro-viscoelastic medium (Biot, 1956c) and nonlinear poroelastic medium (Biot, 1972). Besides, Biot's theory was extended to the unsaturated porous medium by Brutsaert (1964), Berryman et al. (1988), Santos et al. (1990), Wei and Muraleetharan (2002), Hanyga and Lu (2004).

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**Nomenclature**

$A_\alpha$, $\alpha$=s, w, n  the amplitude for the dilatation of the solid phase, the wetting phase and the non-wetting phase for the harmonic plane wave
$A^{(\alpha)}$, $\alpha$=s, w, n  the free energy for the solid phase, the wetting phase and the non-wetting phase, respectively
$\hat{A}^{(\alpha)}$, $\alpha$=s, w, n  the constitutive function for the free energy of the solid phase, the wetting phase and the non-wetting phase
$\mathbf{b}^{(\alpha)}$, $\alpha$=s, w, n  external supply of linear momentum to the solid phase, the wetting phase and the non-wetting phase, respectively
$b_f$, $f$=w, n  parameters accounting for the drag forces between the solid skeleton and the wetting phase as well as the non-wetting phase
$\mathbf{D}$  the fourth order elastic tensor for the solid phase
$\mathbf{E}^{(s)}$  the strain tensor for the solid phase
$e_\alpha$, $\alpha$=s, w, n  dilatation for the solid phase, the wetting phase and the non-wetting phase
$K_s$  the bulk modulus of the solid phase
$K_{cp}$  the modulus accounting for the relation between the capillary pressure and the variation of the saturation of the wetting phase
$K_{uj}$  quasi-unjacketed bulk modulus of the porous medium
$K_{ds}$  the capillary pressure relaxation coefficient for the porous medium
$K_{d\phi}$  the porosity relaxation coefficient for the porous medium
$k$  the intrinsic permeability of the porous medium
$k_r^{(f)}$, $f$=w, n  the relative permeability for the wetting phase and the non-wetting phase
$k_{pi}$, $i$=1, 2, 3  the complex wavenumber for the three compression waves of the porous medium
$k_s$  the complex wavenumber for the shear wave of the porous medium
$\mathbf{L}^{(\alpha)}$, $\alpha$=s, w, n  the gradient of the velocities for the solid phase, the wetting phase and the non-wetting phase
$M_{pp}$  a modulus accounting for the compressibility of the porosity of the porous medium
$M_{ep}$  a modulus accounting for the coupling between the dilation of the solid phase and the variation of the porosity
$M_{ff}$, $f$=w, n  a modulus accounting for the variation of the true density for the wetting phase and the non-wetting phase
$N_{ff}$, $f$=w, n  a modulus accounting for the variation of the volume fraction of the wetting phase and the non-wetting phase
$n^{(\alpha)}$, $\alpha$=s, w, n  the volume fractions for the solid phase, the wetting phase and the non-wetting phase
$p^{(f)}$, $f$=w, n  the pressure for the wetting phase and the non-wetting phase, respectively
$S_0$  the saturation of wetting phase at the initial reference equilibrium state
$S$  the saturation of the wetting phase; for the linear theory, $S$ also denotes the variation of the saturation of the wetting phase
$t$  time
$\bar{\mathbf{T}}^{(\alpha)}$, $\alpha$=s, w, n  the rate of linear momentum transfer to the solid phase, the wetting phase and the non-wetting phase
$\mathbf{u}^{(\alpha)}$, $\alpha$=s, w, n  the displacement for the solid phase, the wetting phase and the non-wetting phase
$\mathbf{v}^{(\alpha)}$, $\alpha$=s, w, n  the velocity for the solid phase, the wetting phase and the non-wetting phase
$v_{pi}$, $i$=1, 2, 3  the phase velocity of the three compression wave of the porous medium
$v_s$  the phase velocity of the shear wave of the porous medium
$\beta_f$, $f$=w, n  fluid content coefficient for the wetting phase and the non-wetting phase
$\phi$  the porosity of the porous medium; for the linear theory, $\phi$ also denotes the variation of the porosity of the porous medium
$\gamma_0^{(f)}$, $f$=w, n  the true density for the wetting phase and the non-wetting phase at the initial reference equilibrium state
$\gamma^{(\alpha)}$, $\alpha$=s, w, n  the true density for the solid phase, the wetting phase and the non-wetting phase
$\Gamma_c$  a parameter accounting for the relation between the capillary pressure and the variation of the saturation of the wetting phase