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The application of finite-difference time-domain modelling for the assessment of GPR in magnetically lossy materials

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ABSTRACT

Numerical modelling has recently established itself as an important, near-surface GPR interpretation tool with the finite-difference, time-domain (FDTD) method becoming one of the most popular techniques. Robust, flexible and accurate, the FDTD technique is capable of simulating GPR wave propagation in complex, three-dimensional, heterogeneous, lossy, subsurface environments to a high degree of realism. Unfortunately, many of the current FDTD methods still consider the subsurface materials as being 'non magnetic' and, as such, do not include the propagation and loss effects associated with magnetic materials (e.g., basic igneous rocks, iron-rich sands, corroded steel reinforced concrete, smelting wastes, etc). For magnetically lossy materials, the inclusion of a complex magnetic permeability into the FDTD scheme can result in smeared or 'fuzzy' interface problems, increased computational demand and equation-level coding changes. Therefore, it is prudent to describe the magnetically derived loss and propagation characteristics in a more generic manner where the 'electric' (e.g., permittivity and conductivity) properties of the material incorporate the magnetic loss effects explicitly. In this paper, we present a "generalised complex effective permittivity" approach to the FDTD material descriptors that allows for the true loss and propagation characteristics of the magnetic materials to modelled fully, regardless of their individual magnetic or electric field relaxation mechanisms. In doing so, we are able to incorporate the lossy, dispersive effects directly into existing FDTD schemes without modification, additional error or increased computational demand. To demonstrate its application, a three-dimensional, 450 MHz, near-surface model of GPR data simulation over a rusty pipe has been included that illustrates how the FDTD modelling can be used to evaluate subtle changes in the spectral nature of the reflected signals. The modelling results show that, for favourable conditions, GPR techniques could be used to provide important, practical information on the assessment of pipeline corrosion and pre-failure conditions.

1. Introduction

The use of numerical modelling tools for the advanced evaluation, interpretation and analysis of ground-penetrating radar (GPR) in complex, heterogeneous, lossy, near-surface environments has become increasingly commonplace in the past few years, particularly with the expansion in cost-effective computing resources. Of all the numerical simulation methods, the Finite Difference Time-Domain (FDTD) approach has established itself as one of the popular techniques as it is able to model realistic and practical GPR scenarios that include accurate antenna models and inhomogeneous, anisotropic and lossy sub-surface materials (Cassidy, 2007b). Near-surface GPR modelling examples have included: land mine detection (Gürel and Oğuz, 2000; Montoya and Smith, 1999), buried tanks and pipes (Zeng and McMechan, 1997; Crocco et al., 2007), contaminated land studies (Cassidy, 2006, 2008), antenna analysis (Bourgeois and Smith, 1996; Roberts and Daniels, 1997; Holliger and Bergmann, 1998; Nishioka et al., 1999; Radzevicius et al., 2003; Lampe and Holliger, 2005; Lampe et al., 2005), dispersive soils (Weedon and Rappaport, 1997; Teixeria et al., 1998), borehole GPR (Holliger and Bergmann, 2002; Wang and McMechan, 2002), NDT applications (Shaari et al., 2004; Giannopoulos, 2005b) and archaeological/forensic features (Hammon et al., 2000). For 'simple' non-dispersive environments (such as dry materials, simple structures, etc), the FDTD formulations have been based on schemes with uniform, lossless materials where the electrical and magnetic properties of the materials are described by specific values of constant (i.e., frequency-independent) relative permittivity (ε_r) or magnetic permeability (μ_r). For more complex environments, where lossy, dispersive materials are present, more advanced FDTD schemes have been developed that include additional variables in the numerical formulation to account for the frequency-dependent

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effect of the material properties (e.g., Cassidy, 2001; Bergmann et al., 1998). However, these schemes still tend to assume that the electromagnetic losses are related to the electrical properties only (i.e., the permittivity and conductivity) and that the magnetic properties of the material are constant, loss-less and, in many cases, equal to the relative free-space value of $\mu_r = 1$. In practice, this is an acceptable assumption for many iron oxide free materials (such as clean siliceous sands/soils, fresh concrete, etc) but if significant quantities of magnetically lossy minerals are present, such as magnetite, hematite, maghemite and/or other iron-rich constituents, then the non-magnetic assumption is invalid. Magnetically lossy materials are more common than we would expect and many subsurface materials do contain significant amounts of magnetic material (e.g., iron-rich soils, basic igneous rocks, smelting waste, corroded reinforced concrete, etc). Consequently, the effect of the magnetic components on the GPR signal attenuation and/or reflection strength can be significant (Olhoeft and Capron, 1993; Van Dam et al., 2002; Cassidy, 2007b) and, therefore, the loss and propagation effects of the magnetically lossy minerals must be included in the FDTD formulations. Unfortunately, the inclusion of a non-constant, frequency-dependent magnetic permeability, where μ_r is effectively replaced by its complex valued equivalent, $\mu_r^* = \mu_r' - j\,\mu_r''$, results in added numerical complexity, modelling error and computational demand in the FDTD formulations. As such, there is need for a simple, yet effective, approach to modelling magnetic effects without the inherent disadvantages of adding a second frequency-dependent material property parameter in to the FDTD formulation.

In this paper, we aim to show that magnetically lossy materials can be included into conventional, dispersive FDTD modelling schemes through the use of a 'generalised complex effective permittivity' approach that incorporates all the electromagnetic loss and propagation effects regardless of whether they are related to magnetic or electric field relaxations. The paper is divided into the following sections: firstly, the theoretical and mathematical basis for the dispersive FDTD schemes will be briefly discussed, including the description of frequency-dependent 'electrical' properties and the numerical complexities associated with including a lossy permeability parameter. After this, the inclusion of the magnetically lossy materials into the generalised complex effective permittivity will be described, along with appropriate examples from real materials. Finally, a representative FDTD example (a 450 MHz GPR survey over a rusting, fuel gas pipe) will be provided to illustrate the application of the modelling approach for the advanced interpretation of near-surface GPR in these magnetically lossy environments.

2. Finite-difference time-domain GPR modelling in dispersive materials

The finite-difference time-domain (FDTD) modelling of ground penetrating radar is essentially the numerical simulation of electromagnetic wave propagation in both time and space as described by the differential form of Maxwell's Equations (Balanis, 1989, 1997). From a mathematical perspective, this represents the solution of the electromagnetic field equations in space/time by approximating the derivative of the electric and magnetic field functions with a difference equation containing a finite spatial and temporal step size (Kunz and Luebbers, 1993). The flexibility of the technique has lead to a wide range of FDTD schemes but they all share common mathematical elements. The full theory and practice of the FDTD method can found in the texts of Taflove and Hagness (2005), Taflove (1995, 1998) and Kunz and Luebbers (1993) and for GPR applications in Cassidy (2001, 2007a) and Giannopoulos (2005a). For GPR, the 'Centralised Leapfrog' FDTD scheme has become the most popular where the components of the electric (E) and magnetic (H) field vectors are interleaved in both time and space in a regimented, repeatable orthogonal, Cartesian (x, y, z) grid of 'nodal' field points (Fig. 1) referred to individually as 'Yee Cells' (Yee, 1966). For a lossless medium with constant frequency independent permittivity (ε) and magnetic permeability (μ), and in the absence of a source, the differential field equations in space (x, y, z) and time, t, are replaced by difference equations in computational space (i, j, k) and temporal increment ($t, t+1, t+2\ldots$) (Kunz and Luebbers, 1993):

$$\frac{\partial Hx}{\partial t} = \frac{1}{\mu}\left(\frac{\partial Ey}{\partial z} - \frac{\partial Ez}{\partial y}\right) \tag{1}$$

$$\frac{\partial Hy}{\partial t} = \frac{1}{\mu}\left(\frac{\partial Ez}{\partial x} - \frac{\partial Ex}{\partial z}\right) \tag{2}$$

$$\frac{\partial Hz}{\partial t} = \frac{1}{\mu}\left(\frac{\partial Ex}{\partial y} - \frac{\partial Ey}{\partial x}\right) \tag{3}$$

$$\frac{\partial Ex}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial Hz}{\partial y} - \frac{\partial Hy}{\partial z}\right) \tag{4}$$

$$\frac{\partial Ey}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial Hx}{\partial z} - \frac{\partial Hz}{\partial x}\right) \tag{5}$$

$$\frac{\partial Ez}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial Hy}{\partial x} - \frac{\partial Hx}{\partial y}\right) \tag{6}$$

$$\frac{Hx^{t+1/2}_{(i,j,k)} - Hx^{t-1/2}_{(i,j,k)}}{\Delta t} = \frac{1}{\mu}\left(\frac{Ey^{t}_{(i,j,k+1)} - Ey^{t}_{(i,j,k)}}{\Delta z} - \frac{Ez^{t}_{(i,j+1,k)} - Ez^{t}_{(i,j,k)}}{\Delta y}\right) \tag{7}$$

$$\frac{Hy^{t+1/2}_{(i,j,k)} - Hy^{t-1/2}_{(i,j,k)}}{\Delta t} = \frac{1}{\mu}\left(\frac{Ez^{t}_{(i+1,j,k)} - Ez^{t}_{(i,j,k)}}{\Delta x} - \frac{Ex^{t}_{(i,j,k+1)} - Ex^{t}_{(i,j,k)}}{\Delta z}\right) \tag{8}$$

$$\frac{Hz^{t+1/2}_{(i,j,k)} - Hz^{t-1/2}_{(i,j,k)}}{\Delta t} = \frac{1}{\mu}\left(\frac{Ex^{t}_{(i,j+1,k)} - Ex^{t}_{(i,j,k)}}{\Delta y} - \frac{Ey^{t}_{(i+1,j,k)} - Ey^{t}_{(i,j,k)}}{\Delta x}\right) \tag{9}$$

$$\frac{Ex^{t+1}_{(i\cdot j\cdot k)} - Ex^{t}_{(i\cdot j\cdot k)}}{\Delta t} = \frac{1}{\varepsilon}\left(\frac{Hz^{t+1/2}_{(i\cdot j+1\cdot k)} - Hz^{t+1/2}_{(i\cdot j\cdot k)}}{\Delta y} - \frac{Hy^{t+1/2}_{(i\cdot j\cdot k+1)} - Hy^{t+1/2}_{(i\cdot j\cdot k)}}{\Delta z}\right) \tag{10}$$

$$\frac{Ey^{t+1}_{(i\cdot j\cdot k)} - Ey^{t}_{(i\cdot j,k)}}{\Delta t} = \frac{1}{\varepsilon}\left(\frac{Hx^{t+1/2}_{(i\cdot j\cdot k+1)} - Hx^{t+1/2}_{(i\cdot j\cdot k)}}{\Delta z} - \frac{Hz^{t+1/2}_{(i+1\cdot j\cdot k)} - Hz^{t+1/2}_{(i\cdot j\cdot k)}}{\Delta x}\right) \tag{11}$$

$$\frac{Ez^{t+1}_{(i\cdot j\cdot k)} - Ez^{t}_{(i\cdot j\cdot k)}}{\Delta t} = \frac{1}{\varepsilon}\left(\frac{Hy^{t+1/2}_{(i+1\cdot j\cdot k)} - Hy^{t+1/2}_{(i\cdot j\cdot k)}}{\Delta x} - \frac{Hx^{t+1/2}_{(i\cdot j+1\cdot k)} - Hx^{t+1/2}_{(i\cdot j\cdot k)}}{\Delta y}\right) \tag{12}$$

where Hx, Hy and Hz are the components of the magnetic field vector, Ex, Ey and Ez are the components of the electric field vector, ε is the permittivity of the subsurface materials and μ is the magnetic permeability. In this basic, second-order accurate formulation the subsurface material properties are described by a constant magnetic permeability (usually assumed to be equal to that of free space) and a constant permittivity (electric effects). Therefore, the formulation does not include loss or conductivity components. To incorporate these, a number of dispersive FDTD schemes have been designed that include the recursive convolution and so-called 'visco-elastic' techniques (e.g., ; Teixeria et al., 1998; Bergmann et al., 1998; Taflove, 1998; Cassidy, 2001; Giannopoulos, 2005a). The schemes are all similar in approach in that they include additional time-dependent variables (sometimes referred to as memory variables) to determine the time