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Reliability of resistivity quantification for shallow subsurface water processes

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ABSTRACT

The reliability of surface-based electrical resistivity tomography (ERT) for quantifying resistivities for shallow subsurface water processes is analysed. A method comprising numerical simulations of water movement in soil and forward–inverse modeling of ERT surveys for two synthetic data sets is presented. Resistivity contrast, e.g. by changing water content, is shown to have large influence on the resistivity quantification. An ensemble and clustering approach is introduced in which ensembles of 50 different inversion models for one data set are created by randomly varying the parameters for a regularisation based inversion routine. The ensemble members are sorted into five clusters of similar models and the mean model for each cluster is computed. Distinguishing persisting features in the mean models from singular artifacts in individual tomograms can improve the interpretation of inversion results.

Especially in the presence of large resistivity contrasts in high sensitivity areas, the quantification of resistivities can be unreliable. The ensemble approach shows that this is an inherent problem present for all models inverted with the regularisation based routine. The results also suggest that the combination of hydrological and electrical modeling might lead to better results.

1. Introduction

The quantification of water content by geophysical methods is an important focus of hydrogeophysical research. Surface based electrical resistivity tomography (ERT) is a promising method, because it is non-intrusive and can cover large surface areas quickly, while it might also be permanently installed for automated monitoring purposes. The development of inversion software for the processing of measured (apparent) resistivities to models of true resistivity has made fast and extensive surveys possible (Daily et al., 2004). Consequently, assessing the reliability of ERT for quantifying soil water content is a currently active research field.

ERT has successfully been used in a number of different applications, e.g. in borehole surveys of tracer experiments (Slater et al., 2000; Kemna et al., 2002) or in laboratory experiments (Binley et al., 1996; Slater et al., 2002). It has also been applied in surface-based surveys of the vadose zone (e.g. Daily and Ramirez, 1992) and of groundwater flow after heavy rain (Suzuki and Higashi, 2001).

Because choice of measurement configuration and inversion parameters may have significant influence on the survey results, improving the quality of ERT surveys has been an intense research topic. Dahlin and Zhou (2004) have compared 10 different electrode arrays for 2D surveys and assessed their quality using synthetic data sets. Stummer et al. (2004) have developed algorithms to calculate optimal electrode arrays that provide as much information on the subsurface as possible. The effects of measurement errors (Zhou and Dahlin, 2003; Oldenborger et al., 2005) and geometry (Loke, 2000; Hennig et al., 2005; Sjoedahl et al., 2006) and inversion parameters (Carle et al., 1999; Rings et al., 2005) on the surveys have been studied.

Geophysical methods cannot directly determine hydrological properties like soil water content. They must be deducted using a general or calibrated relationship between the attribute of interest and the property available through geophysical measurements. In the case of ERT, the resistivities of the subsurface are related to water content by a generic petrophysical relation; usually the equation by Archie (1942). The resistivities, again, are not readily available from surface-based ERT surveys, but must be obtained from the measured apparent resistivities via inversion. The most widespread inversion methods rely on regularised least-squares minimisation to find the smoothest model of resistivities that gives a model response closest to the measured apparent resistivities.

Even assuming that the petrophysical relation between resistivity and water content is known, the resistivity models are non-unique and have likely been affected by the inversion process. The sensitivity of tomographic surveys plays a major role in the retrieval of subsurface characteristics, e.g. for surface-based ERT the sensitivity decreases with depth. Low sensitivity areas (but not only those) can often be plagued by inversion artifacts (e.g. Rings et al., 2008). The inversion process and the choice of inversion parameters, e.g. the regularisation parameters, determine how well the inverted model will reproduce the real distribution. However, some of the parameter

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choices cannot reliably be based upon observation, but must be fitted or depend on experience.

Day-Lewis et al. (2004, 2005) refer to the loss of information caused by the inversion process, lack of sufficient prior information and survey geometry as ‘correlation loss’. They developed a method to compute the correlation loss as a function of the influencing factors. This allows an analytical integration of these factors into geostatistical analyses of quantitative hydrological field surveys, but needs a priori knowledge of covariance models. Singha and Gorelick (2006) suggest a nonstationary estimation approach that uses numerical simulations of transport and electrical current flow to deduct apparent petrophysical relations. These methods modify the translation from the inverted models by adjusting the petrophysical relation but require either a priori knowledge or are computationally intensive.

To assess the quality of ERT-based water content quantification, the complete processing chain including the inversion process, the petrophysical relation and numerical simulations of the soil water movement has to be evaluated. This study introduces a combined approach using soil hydraulic simulations and ensemble building of inverted models to estimate the uncertainty inherent in typical applications of ERT for water content quantification.

2. Methods

To evaluate the inversion process, a forward–inverse cycle approach is used. In numerous applications and studies, forward modeling of synthetic data sets has been used to gain additional insight and confidence into measurements and the inversion process (Loke and Dahlin, 2002; Godio and Naldi, 2003; Hauck and Vonder Muehll, 2003; Loke et al., 2003; Rings et al., 2005; Nguyen et al., 2005, 2007). Forward modeling routines are applied to synthetic data sets obtained from simulations of soil water movement. For two case studies, the approach is used to discuss how slight variations in the soil structure influence the resistivity retrieval, and thereby the water content retrieval.

The second part of the study proposes an ensemble approach which allows an overview of the possible range of inverted models, improves the analysis and enables general assertions about how well a given model can be characterised through the chosen inversion process.

In the following, each methodological step will be shortly introduced, further discussion will illustrate how these steps can be applied to create and analyse two synthetic data sets. Fig. 1 gives an overview of all steps.

The forward–inverse cycle consists of three steps:

(1) *Simulation of water movement in soil*: A model with specific soil structure is generated for numerical simulation of water movement. The movement of a water front, caused by infiltrating rainfall, is simulated over time. Characteristic states of water percolation are identified (starting with a completely dry soil) and a simplified distribution of water content for each state is extracted.
(2) *Generic resistivity model*: A generic resistivity model mirroring the soil structure from (1) is created.
    - For a model representing a dry state (no water content), resistivities are assigned based on typical values known from laboratory measurements and/or literature.
    - For states of water percolation, changes in water content can be calculated using the water content distribution from (1). They can be transferred into resistivity changes by applying a petrophysical relation, e.g. the equation by Archie (1942).
    - A finite-element based forward modeling routine transfers the generic resistivity models into model responses (sets of apparent resistivities) that correspond to the data that would have been recorded by field surveys. Random noise is added to simulate field measuring conditions.
(3) *Resistivity inversion*: The apparent resistivities are inverted using a suitable inversion scheme. The most widespread inversion schemes include smoothness constrained (L2-norm) methods and robust (L1-norm) schemes which are preferable if sharp layer boundaries are present. The forward–inverse cycle is completed by comparing and evaluating the generic and inverted model of resistivities.

The ensemble method comprises two steps:

(1) *Ensemble generation*: For each data set, an ensemble of 50 different inverted models is created by varying the inversion parameters and/or the inversion scheme. The parameter set is chosen randomly from a parameter space constrained to physically meaningful parameter sets.
(2) *Clustering*: A clustering algorithm is used to group similar models of the ensemble. Cluster members can be averaged to simplify the analysis of the ensemble.

2.1. Forward–inverse cycle

The application of this methodology was governed by the available software codes for modeling and inversion. This section discusses how the steps were specifically realised to create and analyse two synthetic data sets.

2.1.1. Simulation of water movement in soil

A numerical simulation of water movement was used to ensure that realistic distributions of water content (and thus resistivity) were used in this study.

If a continuously connected air phase is assumed, the equation of motion for water in soil was given by Richards (1931) as:

$$\frac{\partial}{\partial t}\theta_\mathrm{w} + \nabla \cdot \left[K_\mathrm{w}\left(\nabla \Psi_\mathrm{m} - \varrho_\mathrm{w}\vec{g}\right)\right] \tag{1}$$

with volumetric water content θ_w, hydraulic conductivity K_w, matric potential Ψ_m, density of water ϱ_w and gravitational acceleration $\vec{g}$. To solve Eq. (1) for water content, the material properties have to be given that connect θ_w, K_w and Ψ_m. Usually, the soil water characteristic $\theta_\mathrm{w}(\Psi_\mathrm{m})$ and the conductivity $K_\mathrm{w}(\theta_\mathrm{w})$ are parameterised.

The most widely used parameterisation for the soil water characteristics (van Genuchten, 1980), written in terms of water saturation $S=(\theta-\theta_\mathrm{r})/(\theta_\mathrm{s}-\theta_\mathrm{r})$ with residual water content θ_r, saturated volumetric water content θ_s and hydraulic head $h_\mathrm{m}=\Psi_\mathrm{m}/(\varrho_\mathrm{w}g)$, is

$$S(h_\mathrm{m}) = \left[1 + (\alpha h_\mathrm{m})^{\nu}\right]^{-1+1/\nu} \tag{2}$$

with the scaling factor α, which is related to the air-entry value $1/\alpha$, and the parameter ν connected to the pore size distribution. The hydraulic conductivity is characterised by applying the parameterisations of Mualem (1976). A concise overview of the soil physics is given e.g. by Stephens (1996).

Eq. (1) was solved numerically using the HYDRUS software (Simunek et al., 2006). By defining time-variable precipitation and evaporation rates as atmospheric boundary conditions, changes in the hydraulic head h_m and thus water movement are induced.

The simulations were conducted with models representing a two-layered soil representative of a site used in previous field studies (Rings et al., 2008). In addition to an atmospheric boundary, a seepage boundary on the bottom allowed water to leave the domain. From the simulations, characteristic states of a water front infiltrating the domain were identified. Generally, beyond the dry state, characteristic states should be chosen at times when the water content distribution