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Letter

On the Berdichevsky average

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ABSTRACT

Through a large number of magnetotelluric (MT) observations conducted in a study area, one can obtain regional one-dimensional (1-D) features of the subsurface electrical conductivity structure simply by taking the geometric average of determinant invariants of observed impedances. This method was proposed by Berdichevsky and coworkers, which is based on the expectation that distortion effects due to near-surface electrical heterogeneities will be statistically smoothed out. A good estimation of a regional mean 1-D model is useful, especially in recent years, to be used as *a priori* (or a starting) model in 3-D inversion. However, the original theory was derived before the establishment of the present knowledge on galvanic distortion. This paper, therefore, reexamines the meaning of the Berdichevsky average by using the conventional formulation of galvanic distortion. A simple derivation shows that the determinant invariant of distorted impedance and its Berdichevsky average is always downward biased by the distortion parameters of shear and splitting. This means that the regional mean 1-D model obtained from the Berdichevsky average tends to be more conductive. As an alternative rotational invariant, the sum of the squared elements (ssq) invariant is found to be less affected by bias from distortion parameters; thus, we conclude that its geometric average would be more suitable for estimating the regional structure. We find that the combination of determinant and ssq invariants provides parameters useful in dealing with a set of distorted MT impedances.

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1. Introduction

The magnetotelluric (MT) method is one of the geophysical exploration methods for studying the electrical conductivity distribution within the Earth. This method estimates the MT impedance tensor, defined as a 2×2 complex-valued tensor relating the observed horizontal vectors of electric and magnetic field fluctuations, and then inverts a set of MT impedances into a conductivity model. Since the establishment of the basic theory (Cagniard, 1953), there has been considerable progress in the technology of inversion of MT impedances (e.g., Ogawa, 2002; Siripunvaraporn, 2012). Three-dimensional (3-D) inversion is now possible, at least to some extent, in complex and realistic situations where target anomalous bodies of various scales are distributed in the Earth, if a number of observations are made over a sufficiently wide area including the largest scale with a typical site spacing to resolve the target of smallest scale. However, in practice, because the number of possible observations is limited and the underground structure is unknown prior to exploration, it is often difficult to design an MT observation array so as to satisfy this ideal condition. In many cases, the site spacing has to be designed larger than the typical scale of near-surface lateral heterogeneities, especially in an area of complex surface geology (Fig. 1). This causes spatial aliasing in the sampling of spatially heterogeneous electromagnetic fields and thus of the MT impedance. We shall hereafter refer to aliasing of the MT impedance as “distortion” and the treatment of galvanic distortion in 3-D MT inversion is our main concern.

In theory, the distortion of the MT impedance may be both galvanic and inductive, but we can choose a proper frequency range so that near-surface lateral heterogeneities have only galvanic effects (Utada and Munekane, 2000). This paper treats such a case. Galvanic distortion is still a kind of spatial aliasing and hence must be removed independently before a reliable image of the underground structure can be obtained through the inversion of MT impedances (Simpson and Bahr, 2005). The removal of galvanic distortion is possible and practical assuming the regional structure is 2-D (Groom and Bailey, 1989); no generally practical method has yet been established for the 3-D case (Sasaki and Meju, 2006; Jones, 2011). The use of impedance properties unaffected by galvanic distortion such as the phase tensor (Caldwell

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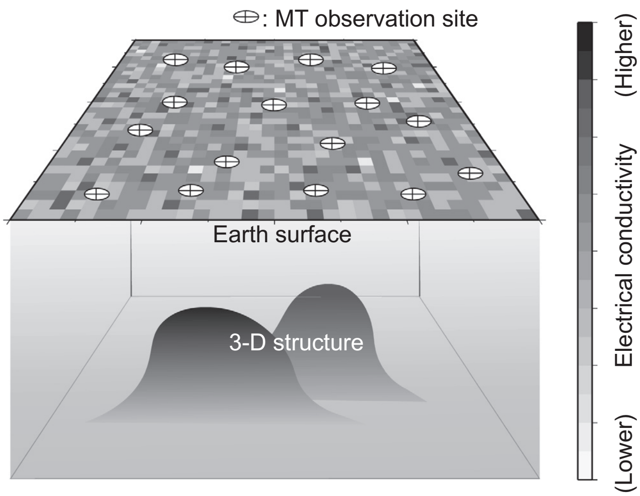

Fig. 1. A sketch of MT array study for 3-D electrical conductivity structure considered in this paper. Observation sites are located in a study region with complex surface geology. If near-surface heterogeneities with spatial scales smaller than the typical site spacing have significant galvanic effects, these effects are aliased to cause galvanic distortion.

et al., 2004) is another option to avoid the difficulty of removing galvanic distortion in the 3-D scenario. However, inversion of the phase tensor is more dependent on the starting model as a natural consequence of dealing with less information (Patro et al., 2013).

Berdichevsky et al. (1980) proposed a simple scheme to obtain the regional MT impedance by averaging the impedances measured at a number of stations in one study area via the equation

$$\log \bar{Z}_{det}(\omega) = \frac{1}{N}\sum_{i=1}^{N} \log Z_{det}(\mathbf{r}_i; \omega), \tag{1}$$

where $\mathbf{r}_i$ and ω are the location of the i-th site and angular frequency, respectively, and Z_{det} is what the authors called the effective impedance defined by the determinant of the impedance tensor.

$$Z_{det} = \sqrt{Z_{xx}Z_{yy} - Z_{xy}Z_{yx}}. \tag{2}$$

In this paper, we will call $Z_{\det}$ as the determinant impedance.

Berdichevsky et al. (1980) further assumed Z_{det} in Eq. (1) to be expressible as a product of normal (site-independent) impedance, Z_N, and local (site-dependent) distortion coefficient, K, at each observation site as

$$Z_{det}(\mathbf{r}_i; \omega) = K(\mathbf{r}_i) \cdot Z_N(\omega). \tag{3}$$

Using magnetotelluric results in the Baikal region, Berdichevsky et al. (1980) showed that the distribution of effective impedances can be approximated fairly accurately by the log-normal law. This suggests that the local galvanic effect, modeled using the coefficient K, is a random phenomenon, so that the right-hand side of Eq. (1) becomes

$$\log \bar{Z}_{det}(\omega) = \log Z_N(\omega) + \frac{1}{N}\sum_{i=1}^{N} \log K(\mathbf{r}_i) \approx \log Z_N(\omega). \tag{1'}$$

Similar procedures have been often employed, particularly by scientists in the former Soviet Union (e.g., Berdichevsky et al., 1989) and even in recent works to estimate the regional response and thus, we shall call the average of Eqs. (1) or (1′) the "Berdichevsky average".

Suppose a situation in which we try to study 3-D electrical conductivity distribution by inverting a set of MT impedances obtained by a given observation array and MT impedances include galvanic distortion (Fig. 1). We define a regional mean 1-D conductivity profile, $\sigma_R(z)$, by a surface integral in a area, ΔS, which the observation array occupies,

$$\sigma_R(z) = \frac{1}{\Delta S}\iint_{\Delta S} \sigma(x, y, z)dxdy, \tag{4}$$

where $\sigma(x,y,z)$ is 3-D electrical conductivity distribution in the Earth. In general, $\sigma(x,y,z)$ can be expressed by a sum of mean 1-D profile and the lateral heterogeneity or the conductivity anomaly, $\delta\sigma(x,y,z)$, as,

$$\sigma(x, y, z) = \sigma_R(z) + \delta\sigma(x, y, z). \tag{5}$$

These equations clearly show a merit of using $\sigma_R(z)$ as *a priori* (or a starting) model for 3-D inversion in a sense that it minimizes the variance of lateral heterogeneity, $\delta\sigma$. Alternatively, one can use logarithmic average,

$$log\sigma_R(z) = \frac{1}{\Delta S}\iint_{\Delta S} \log \sigma(x, y, z)dxdy, \tag{4'}$$

if the variance is to be minimized in log-scale. A further remaining question is how to obtain a good estimate of $\sigma_R(z)$ before conducting 3-D inversion from a set of MT impedances with galvanic distortion. The Berdichevsky average has been regarded as one of the most practical solutions (e.g. Tada et al., 2014; Avdeeva et al., 2015).

Although the distortion coefficient K in Eq. (1′) was treated as a real-valued scalar quantity without a clear definition except being a random quantity, present knowledge tells us that, in general, it should be derived from a 2×2 tensor quantity describing galvanic distortion acting on the regional impedance. In this paper, we examine the meaning of the Berdichevsky average given by Eq. (1′) by using the expression for galvanic distortion by Groom and Bailey (1989).

2. Determinant invariant of the MT impedance with galvanic distortion

We start from a general expression for the impedance tensor, $\mathbf{Z}$, with galvanic distortion at each observation site (Fig. 1), whose location is $\mathbf{r}_i$, distributed over our study area (Groom and Bailey, 1989; Utada and Munekane, 2000; Bibby et al., 2005):

$$\mathbf{Z}(\mathbf{r}_i; \omega) = \mathbf{C}(\mathbf{r}_i) \cdot \mathbf{Z}_R(\mathbf{r}_i; \omega), \tag{6}$$

where $\mathbf{C}$ is a 2×2 real-valued tensor describing galvanic distortion and $\mathbf{Z}_R$ is the so-called regional (undistorted) impedance, which reflects the induction effects due to the regional conductivity structure. Here, we treat a general 3-D regional structure so that $\mathbf{Z}_R$ is a 2×2 full complex-valued tensor.

Following Groom and Bailey (1989), the galvanic distortion tensor, $\mathbf{C}$, is composed of two elements:

$$\mathbf{C}(\mathbf{r}_i) = g_i \cdot \mathbf{D}(\mathbf{r}, t_i, e_i, s_i). \tag{7}$$

Here, g_i is a scalar parameter reflecting the impedance amplitude at each site due to local effect, and is thus referred to as static shift or site gain. $\mathbf{D}$ is a 2×2 real-valued tensor describing geometric distortion (twisting, shearing, and splitting) with respective scalar parameters t_i, e_i, and s_i at each site (Groom and Bailey, 1989).

The tensor $\mathbf{D}$ is normalized so that the Frobenius norm of $\mathbf{C}$ is equal to $\sqrt{2}g_i$ (Bibby et al., 2005), and therefore following Groom and Baily (1989)'s notation, we derive,

$$\mathbf{D}(\mathbf{r}_i, t_i, e_i, s_i) = N_i \begin{pmatrix} (1+s_i)(1-t_ie_i) & (1-s_i)(e_i-t_i) \\ (1+s_i)(e_i+t_i) & (1-s_i)(1+t_ie_i) \end{pmatrix}, \tag{8}$$

where

$$N_i = \frac{1}{\sqrt{1+t_i^2}\sqrt{1+e_i^2}\sqrt{1+s_i^2}}. \tag{9}$$

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