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Double point source W-phase inversion: Real-time implementation and automated model selection

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ABSTRACT

Rapid and accurate characterization of an earthquake source is an extremely important and ever evolving field of research. Within this field, source inversion of the W-phase has recently been shown to be an effective technique, which can be efficiently implemented in real-time. An extension to the W-phase source inversion is presented in which two point sources are derived to better characterize complex earthquakes. A single source inversion followed by a double point source inversion with centroid locations fixed at the single source solution location can be efficiently run as part of earthquake monitoring network operational procedures. In order to determine the most appropriate solution, i.e., whether an earthquake is most appropriately described by a single source or a double source, an Akaike information criterion (AIC) test is performed. Analyses of all earthquakes of magnitude 7.5 and greater occurring since January 2000 were performed with extended analyses of the September 29, 2009 magnitude 8.1 Samoa earthquake and the April 19, 2014 magnitude 7.5 Papua New Guinea earthquake. The AIC test is shown to be able to accurately select the most appropriate model and the selected W-phase inversion is shown to yield reliable solutions that match published analyses of the same events.

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1. Introduction

The W-phase is a long period (up to 1000 s) phase of a seismic source that arrives between the P- and S-wave phases. Kanamori and Rivera (2008) were the first to use the W-phase in an accurate method for assessing source properties of great earthquakes, i.e., earthquakes with magnitudes of at least M_w 8.0. More recently, source inversion of the W-phase has been extended to earthquakes with much lower magnitudes, and has been implemented in real-time by the National Earthquake Information Center (NEIC) of the U.S. Geological Survey (USGS), the Pacific Tsunami Warning Center (PTWC) of the National Oceanic and Atmospheric Administration (NOAA), and the Institut de Physique du Globe de Strasbourg (IPGS) (Hayes et al., 2009; Duputel et al., 2011). Although multiple authors (Duputel et al., 2012; Lay et al., 2013b) have demonstrated that multiple point source inversion using the W-phase is able to obtain accurate representations of events, the possible real-time applicability of a multiple source W-phase inversion has not been explored. Here the W-phase source inversion is extended from the original approach that parameterized all earthquakes as point sources, to allow for a two point source solution. Fixing the centroid locations of the two sub-events and performing a grid search for the corresponding time delays can quickly obtain an accurate representation of a complex event. Once solutions have been found using both the traditional single source and the new double source W-phase inversions, an AIC test is performed to select the model that best represents the recorded data. Here we assess the performance of the double point source approach and show that not only are accurate results obtained in near-real-time but also the AIC test can be successfully used to select the best model.

2. Double point source W-phase methods

The double point source W-phase inversion involves four main stages. In the first stage, a single point source W-phase inversion is performed using the NEIC Preliminary Determinations of Epicenters (PDE) hypocenter. An initial magnitude estimate, obtained from either NEIC preliminary evaluations or from a NOAA tsunami warning center, is used to establish the appropriate filter; see Duputel et al. (2012b) and Table 1. The W-phase inversion for a single point source can be formulated as,

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Table 1
Corner frequencies used for Butterworth bandpass filtering in the W-phase inversions. Corner frequencies are selected based on initial magnitude estimates.

Magnitude range (M_w)	Low corner (Hz) (s)	High corner (Hz) (s)
$M_w \geqslant 8.0$	0.001(1000 s)	0.005(200 s)
$8.0 > M_w \geqslant 7.5$	0.002(500 s)	0.0067(150 s)
$7.5 > M_w \geqslant 7.0$	0.002(500 s)	0.0083(120 s)
$7.0 > M_w \geqslant 6.5$	0.0025(400 s)	0.01(100 s)
$6.5 > M_w$	0.0067(150 s)	0.02(50 s)

$$\begin{bmatrix} u_1^{1,1} & u_1^{2,2} & \cdots & u_1^{2,3} \\ u_2^{1,1} & u_2^{2,2} & \cdots & u_2^{2,3} \\ \vdots & \vdots & \cdots & \vdots \\ u_N^{1,1} & u_N^{2,2} & \cdots & u_N^{2,3} \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{22} \\ M_{33} \\ M_{12} \\ M_{13} \\ M_{23} \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix}, \tag{1}$$

where M_{ij} is element *i-j* of the source deviatoric moment tensor, $u_k^{i,j}$ is the displacement at station *k* computed for a moment tensor with only $M_{ij} = 1$ (i.e., the Green's functions), and d_k is the observed W-phase at station *k*. The inversion is performed using the least-squares technique. For more information on the details of the single source W-phase inversion, we refer you to Kanamori and Rivera (2008) and Hayes et al. (2009).

Once the single source solution has been obtained, the double point source W-phase inversion is performed using the centroid parameters of the single source inversion as a starting point. The centroid locations of both sub-events being considered in the double point source inversion are fixed at the centroid location found for the single source solution. Since centroid locations are derived using a least squares grid search approach, fixing the centroid locations greatly speeds up the run time of the inversion while still providing an accurate characterization of the two sub-events. In the third stage, a grid search is applied to find the time shift pair that minimizes the root mean square (RMS) error of the waveform misfit. During this step, both the time delays and the half-durations of the two sub-events are found within the grid search. While performing the grid search, a time delay and half-duration pair is only considered if applying the pair results in the first sub-event ending after the second sub-event begins and prior to the end of the second sub-event, and the second sub-event beginning after the first sub-event begins. Additionally, the half-durations are constrained based on the magnitude of the single source solution so that the half-durations selected for the two sub-events cannot exceed the half-duration of the single source solution and cannot be smaller than the half-duration of a M_W 7.0 event. The half-durations are constrained in such a manner since the two sub-events combined should be equivalent to the single source solution and, since only events of M_W 7.5 and greater are considered here, the two sub-events are not expected to have magnitudes much smaller than M_w 7.0. The reference half-duration formula given by Duputel et al. (2013) was used to calculate the maximum and minimum half-durations. If two time delay and half-duration pairs are found with the same minimum RMS, the solution with the earlier time delays is selected.

A second inversion is then performed using the optimized time shift pair. The final stage compares the single source model and the double point source model using Akaike's method to determine the most appropriate model for the event (Akaike, 1972). The AIC test uses the number of degrees of freedom of each model along with their errors to evaluate the relative fit of each solution. Since we are comparing two models, we are interested in the difference between the AIC values of the models, which is given by,

$$\Delta\mathrm{AIC} = N \times \ln\left(\frac{SS2}{SS1}\right) + 2\Delta\mathrm{df},$$

where *N* is the length of the data, *SS*1 is the sum-of-squares error for the single source inversion, *SS*2 is the sum-of-squares error for the double source inversion, and Δdf is the difference in degrees of freedom between the two models. In this case, since a deviatoric moment tensor is used, the single source inversion has 5 degrees of freedom and the double source inversion has 10 degrees of freedom. If ΔAIC is a negative value, the double source model should be selected, otherwise the single source model is the better choice.

An additional benefit of using the difference between AIC values is the ability to easily compute the likelihood of a model for the given data. For each event, the Akaike weight for the double source model can be calculated as

$$w_{ds} = \frac{e^{-0.5\Delta\mathrm{AIC}}}{\left(e^{-0.5\Delta\mathrm{AIC}} + e^{-0.5\times 0}\right)}.$$

The Akaike weight for the single source model can be written in a similar manner as

$$w_{ss} = \frac{e^{-0.5\times 0}}{\left(e^{-0.5\Delta\mathrm{AIC}} + e^{-0.5\times 0}\right)}.$$

The weights, which sum to one, can be interpreted as the estimated probability that either the single or double source model is better. For example, if $w_{ds} = 0.93$ and $w_{ss} = 0.07$ the double source model is clearly the better representation of the event under consideration. For more information on Akaike's method or Akaike weights, we refer you to Akaike (1972), Akaike (1974), GraphPad (1994-2014), Burnham and Anderson (1998), and Canham (2013).

When a double point source inversion is performed, two sub-events are being inverted simultaneously. The W-phase inversion for a two point source event can therefore be formulated as an extended version of Eq. (1),

$$\begin{bmatrix} u_{1,1}^{1,1} & \cdots & u_{1,1}^{2,3} & u_{2,1}^{1,1} & \cdots & u_{2,1}^{2,3} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ u_{1,N}^{1,1} & \cdots & u_{1,N}^{2,3} & u_{2,N}^{1,1} & \cdots & u_{2,N}^{2,3} \end{bmatrix} \begin{bmatrix} M_{11}^1 \\ M_{22}^1 \\ M_{33}^1 \\ M_{12}^1 \\ M_{13}^1 \\ M_{23}^1 \\ M_{11}^2 \\ M_{22}^2 \\ M_{33}^2 \\ M_{12}^2 \\ M_{13}^2 \\ M_{23}^2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix}.$$

Here, M_{ij}^1 is element *i-j* of the source deviatoric moment tensor for the first event and M_{ij}^2 is element *i-j* of the source deviatoric moment tensor for the second event. Similarly, $u_{1,k}^{i,j}$ is the displacement at station *k* computed for a moment tensor with only $M_{ij}^1 = 1$ while $u_{2,k}^{i,j}$ is the displacement at station *k* computed for a moment tensor with only $M_{ij}^2 = 1$. The observed W-phase at station *k* is still given by d_k, being a superposition of the two sub-events. As in the single point source case, the least-squares technique is used to perform the inversion.

If the double point source model is selected by the AIC test, an extended double point source W-phase inversion can be run to find the optimized centroid locations for the two sub-events. In order to find the centroid locations another grid search would be applied following the inversion using the optimized time shift pair. The centroid grid search performs a preliminary depth grid search