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Waves and linear stability of magnetoconvection in a rotating cylindrical annulus

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ABSTRACT

Magnetohydrodynamic (MHD) waves in a rapidly rotating planetary core can cause the magnetic secular variation. To strengthen our understanding of the physical basis, we revisit the linear stability analyses of thermal convection in a quasi-geostrophic rotating cylindrical annulus with an applied toroidal magnetic field, and we extend the investigation of the oscillatory modes to a broader range of the parameters. Particular attention is paid to influence of thermal boundary conditions, either fixed temperature or heat-flux conditions. While the non-dissipative approximation yields a slow wave propagating retrograde, termed as a Magnetic–Coriolis (MC) Rossby wave, dissipative effects produce a variety of waves. When magnetic diffusion is stronger than thermal diffusion, this can cause a very slow wave propagating prograde. Retrograde-traveling slow waves appear when magnetic diffusion is weaker. Emergence of the slow modes allows convection to occur at lower critical Rayleigh numbers than in the nonmagnetic case. When magnetic diffusion is strong, the onset of the convection occurs with the prograde-propagating slow wave, whereas when it is weak, a slow MC-Rossby mode yields the critical convection. Fixed heat-flux boundary conditions have profound effects on the marginal curves, which monotonically increase with the azimuthal wavenumber, and favor larger length scales at the onset of the convection, provided there is sufficient field strength that the Coriolis force is balanced with the Lorentz force. The effect, however, becomes less clear as magnetic diffusion is weakened and various MHD waves emerge.

1. Introduction

The geomagnetic field has significantly changed over several timescales. Its westward drift, with timescales of a few hundred years, is a significant and well-known feature (e.g. Bullard et al., 1950; Yukutake, 1962; Finlay and Jackson, 2003). Dominant eastward motions, on the other hand, have also been reported to appear in longer time windows, as obtained from archaeological geomagnetic data (e.g. Aitken et al., 1964; Dumberry and Finlay, 2007). To account for the longitudinal geomagnetic drifts, two end-member mechanisms have been proposed [see Holme (2007) and Finlay et al. (2010) for reviews]. One is advection of the magnetic field lines due to large-scale flows; in particular, those near the top of the core (e.g. Bullard et al., 1950). The other mechanism is propagation of the rotating MHD waves that are excited in the deep convective region of the core (Hide, 1966; Malkus, 1967; Canet et al., 2014) or in a stably stratified layer (Braginsky, 1984, 1999), which may potentially be present beneath the core surface. Recent geodynamo simulations in which there are no stably stratified layers have successfully reproduced regional features of the westward drift (e.g. Aubert et al., 2013). Some authors have compared the magnetic drifts and mean zonal flow speeds, and reported that wave propagation would take part in the magnetic drifts (e.g. Kono and Roberts, 2002; Christensen and Olson, 2003). However, the properties of these waves have not been analyzed, and they can be mixtures of several oscillatory modes. The present study aims to clarify what types of waves could be excited in such systems and to assess to what extent they could play a role in the geomagnetic drifts.

Studies of magnetoconvection in rotating spheres and thick spherical shells have shown various oscillatory modes and crucial roles of magnetic fields [e.g. Fearn (1979a,b); see also a review by Jones (2007)]. In the presence of a magnetic field, the Lorentz force can balance the Coriolis force, and this diminishes the frequency of the oscillation and even changes the direction of propagation, from eastward to westward propagation. However, complexities in spherical systems can make it harder to understand the basic oscillatory modes.

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In order to gain the physical basics for classification of modes, a quasi-geostrophic cylindrical annulus model is a useful tool, which is a simplified model of a rotating spherical system and is capable of reproducing the essential features of the nonmagnetic convection (Busse, 1970, 1986). A few authors have applied large-scale external magnetic fields in order to investigate the magnetic oscillatory modes. While Hide (1966) considered the non-dissipative case and found topographic Magnetic-Coriolis waves (MC-Rossby waves) propagating westward, Busse (1976) and Soward (1979) developed the annulus model to analyze the dissipative effects. Busse (1976) obtained the dispersion relations of an MC-Rossby mode modified by diffusion and of another magnetic Coriolis mode, sometimes referred to as a magneto-Rossby wave, and pointed out that both of the modes traveled eastward for strong magnetic diffusion. In the limit of strong magnetic diffusion, the analysis was extended to examine the marginal curves for the onset of the convection and the weakly supercritical convection, respectively, by Busse and Finocchi (1993) and Petry et al. (1997). More recently, Finlay (2008) used approximated dispersion relations to classify a variety of waves in a cylindrical annulus. However, the classification was not applied to the properties of the convection associated with the waves. Since each oscillatory mode leads to a different thermal instability, a comparison of the marginal curves enables us to assess the importance of each oscillatory mode. To complete the classification of the modes, we explore the marginal convection, as well as the wave properties, over a broader range of fundamental parameters. As shown below, the most preferred mode varies with the parameters that are assumed; typically, it is a thermal magneto-Rossby mode when magnetic diffusion is strong, and it is an MC-Rossby mode when magnetic diffusion is weak. These analyses may also contribute to the better understanding of laboratory experiments and related nonlinear simulations (Gillet et al., 2007).

Such convection can be altered by either fixed temperature or fixed heat-flux boundary conditions, even if they are homogeneous. In the classic Rayleigh-Bénard convection, when neither rotation nor a magnetic field is present, constant flux conditions elongate convective cells and favor the longest length scale in the system (e.g. Jakeman, 1968; Busse and Riahi, 1980; Chapman and Proctor, 1980; Ishiwatari et al., 1994). The presence of rapid rotation tends to decrease the length scale and competes with the effect of heat-flux boundary conditions; the result is that the boundary effects diminish as the rotation rate is increased (Riahi, 1982; Takehiro et al., 2002; Gibbons et al., 2007). Provided that a magnetic field has a proper morphology, it can balance the Lorentz and Coriolis forces and reinstate the boundary effect, i.e., prefer the very long length scale (Hori et al., 2012). Although the previous study showed the effect on stationary convection in a plane layer model, investigation of oscillatory modes is of primal importance in spherical systems, where the topography essentially produces the convection which drifts in the azimuthal direction. Adopting the cylindrical annulus model, we shall show that, when the Lorentz force is in balance with the Coriolis force, fixed heat-flux boundary conditions favor larger length scales at the onset of the convection, but the effect becomes less clear as magnetic diffusion is weakened. Analysis of rotating magnetoconvection provides a framework for interpreting the fundamental effects of the thermal boundary conditions on convection-driven MHD dynamos (e.g. Busse and Simitev, 2006; Sakuraba and Roberts, 2009; Hori et al., 2010).

The model setup is described in Section 2. We begin our linear stability analyses in Section 3 by revisiting the case with fixed temperature boundary conditions, and in Section 4 we report the effects of fixed heat-flux boundary conditions. In Section 5 the results are summarized and the geophysical implications are discussed.

2. Model

We consider a Boussinesq fluid in a cylindrical annulus with sloping top and bottom boundaries (e.g. Busse, 1970, 1986; Takehiro et al., 2002). In the literature, there are partial analyses of extensions to the cases where magnetic fields are applied (e.g. Busse, 1976; Soward, 1979; Busse and Finocchi, 1993; Yoshida and Hamano, 1993; Finlay, 2008), and, for clarity, we adopt the same setup. The annulus rotates with angular velocity Ω around the z-axis, and has constant thickness D in the x-direction, has almost constant height L in the z-direction, where the top and bottom boundaries are inclined at a small angle η. The gravity g_0 and applied temperature gradient β are both uniform and antiparallel with the x-axis. A basic magnetic field is externally applied in the y-direction, perpendicular to both the rotational axis and gravity: it is given as

$$\boldsymbol{B}_0 = \hat{\boldsymbol{e}}_y B_{0y}(z) \tag{1}$$

with $\hat{\boldsymbol{e}}_y$ being the unit vector in the y-direction. We thus consider small perturbations in the velocity $\boldsymbol{u}$, the magnetic field $\boldsymbol{b}$, and the temperature θ of the static state.

We assume that the motion is almost geostrophic and that the z-component of the velocity is small compared to the x- and y-components, so that the solutions can be written in a two-dimensional form of

$$\boldsymbol{u} = \nabla \times \psi(x,y)\hat{\boldsymbol{e}}_z, \quad \boldsymbol{b} = \nabla \times g(x,y)\hat{\boldsymbol{e}}_z, \quad \theta = \theta(x,y), \tag{2}$$

where $\hat{\boldsymbol{e}}_z$ is the unit vector in the z-direction. Here $-\nabla^2\psi$ and $-(\nabla^2 g)/\mu$ represent the z-components of the vorticity and the electric current density, respectively, with μ being the magnetic permeability. Averaging over z and applying the sloping boundary conditions, we obtain the linearized equation for the vorticity as

$$\left[\left(\frac{\partial}{\partial t} - \nu\Delta_2\right)\Delta_2 - \frac{4\Omega\eta D}{L}\frac{\partial}{\partial y}\right]\psi = \alpha g_0 \frac{\partial\theta}{\partial y} + \frac{\langle B_{0y}\rangle}{\rho\mu}\frac{\partial}{\partial y}\Delta_2 g, \tag{3}$$

(Busse, 1976; Soward, 1979; Yoshida and Hamano, 1993), where $\Delta_2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, α is the thermal expansivity, ν the kinematic viscosity, ρ the density, and $\langle B_{0y}\rangle$ the mean strength of the imposed field

$$\langle B_{0y}\rangle = \frac{1}{L}\int_{-L/2}^{L/2} B_{0y}(z)dz = \int_{-1/2}^{1/2} B_{0y}(z')dz'. \tag{4}$$

The equations for the z-averaged electric current and the temperature are, respectively, written as

$$\left(\frac{\partial}{\partial t} - \lambda\Delta_2\right)\Delta_2 g = \langle B_{0y}\rangle \frac{\partial}{\partial y}\Delta_2\psi \tag{5}$$

and

$$\left(\frac{\partial}{\partial t} - \kappa\Delta_2\right)\theta = \beta\frac{\partial\psi}{\partial y}, \tag{6}$$

where λ is the magnetic diffusivity and κ the thermal diffusivity. Note that the applied field in the present model is not necessarily uniform in the z-direction (corresponding to the axial direction in a spherical system), but it is uniform in the y-direction (azimuthally uniform or axisymmetric).

We choose the layer thickness D as the length scale, the viscous diffusion time D^2/ν as the time scale, the mean strength of the applied magnetic field $\langle B_{0y}\rangle$ as the magnetic scale, and βD as the temperature scale. The governing equations are rewritten in a dimensionless form as

$$\left[\left(\frac{\partial}{\partial t_*} - \Delta_{2*}\right)\Delta_{2*} - \eta^*\frac{\partial}{\partial y_*}\right]\psi_* = \frac{Ra}{Pr}\frac{\partial\theta_*}{\partial y_*} + \frac{Q}{Pm}\frac{\partial}{\partial y_*}\Delta_{2*}g_*, \tag{7}$$