Physics of the Earth and Planetary Interiors 241 (2015) 15–20

Contents lists available at ScienceDirect

Physics of the Earth and Planetary Interiors

journal homepage: www.elsevier.com/locate/pepi

Influence of surface displacement on solid state flow induced by horizontally heterogeneous Joule heating in the inner core of the Earth

Shin-ichi Takehiro

Research Institute for Mathematical Sciences, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan

A R T I C L E   I N F O

Article history:
Received 25 September 2014
Received in revised form 11 January 2015
Accepted 9 February 2015
Available online 14 February 2015

Keywords:
Inner core flows
Elastic anisotropy
Turbulent velocity in the outer core
Interaction between inner and outer core

A B S T R A C T

We investigate the influence of surface displacement on fluid motions induced by horizontally heterogeneous Joule heating in the inner core. The difference between the governing equations and those of Takehiro (2011) is the boundary conditions at the inner core boundary (ICB). The temperature disturbance at the ICB coincides with the melting temperature, which varies depending on the surface displacement. The normal component of stress equalizes with the buoyancy induced by the surface displacement. The toroidal magnetic field and surface displacement with the horizontal structure of Y_2^0 spherical harmonics is given. The flow fields are calculated numerically for various amplitudes of surface displacement with the expected values of the parameters of the core. Further, by considering the heat balance at the ICB, the surface displacement amplitude is related to the turbulent velocity amplitude in the outer core, near the ICB. The results show that when the turbulent velocity is on the order of 10^{-1}–10^{-2} m/s, the flow and stress fields are similar to those of Takehiro (2011), where the surface displacement vanishes. As the amplitude of the turbulent velocity decreases, the amplitude of the surface displacement increases, and counter flows from the polar to equatorial regions emerge around the ICB, while flow in the inner regions is directed from the equatorial to polar regions, and the non-zero radial component of velocity at the ICB remains. When the turbulent velocity is on the order of 10^{-4}–10^{-5} m/s, the radial component of velocity at the ICB vanishes, the surface counter flows become stronger than the flow in the inner region, and the amplitude of the stress field near the ICB dominates the inner region, which might be unsuitable for explaining the elastic anisotropy in the inner core.

1. Introduction

The origin of the elastic anisotropy of the Earth's inner core (e.g. Poupinet et al., 1983; Morelli et al., 1986; Souriau, 2007) is considered to be the alignment of texture formed along the solidification of the core (e.g. Karato, 1993; Bergman, 1997) or the alignment of the preferred orientation of crystals by plastic deformation of fluid motions (e.g. Jeanloz and Wenk, 1988; Yoshida et al., 1996; Karato, 1999; Buffett and Wenk, 2001). The depth dependency of the anisotropy is difficult to explain by the solidification mechanism, whereas the various factors driving solid state flow in the inner core considered thus far do not appear to yield sufficiently strong stresses to generate elastic anisotropy. Takehiro (2011) proposed Joule heating of the magnetic field penetrating diffusively from the inner core boundary (ICB) as a possible source of inner core flows. His specific calculation in the case of a toroidal magnetic field with the horizontal structure of Y_2^0 spherical harmonics showed that internal flows of sufficient magnitude can be induced to explain the elastic anisotropy. The obtained solution consists of downward flow in the equatorial region and upward flows in the polar region, and has a non-zero radial velocity component at the ICB, causing mass exchange between the inner and outer core. This feature is a result of the constant normal stress boundary condition at the ICB, and it is implicitly assumed that the phase change occurs instantaneously at the ICB. However, the actual speed of the phase change is finite. If the speed of the phase change is slow enough, the ICB would be deformed, and surface displacement is induced by the non-zero radial velocity at the ICB. This surface displacement may prevent inner core flows due to the buoyancy force originating from the density contrast between the inner and outer core.

In this paper, we investigate the influence of surface displacement on fluid motions induced by horizontally heterogeneous Joule heating in the inner core. We examine the extent of development of surface displacement, and modification of the flow field of the inner core. Section 2 is a description of our model. In Section 3, numerical results are presented for various amplitudes of surface

E-mail address: takepiro@gfd-dennou.org

http://dx.doi.org/10.1016/j.pepi.2015.02.003
0031-9201/

displacement at the ICB. Further, the equilibrated amplitude of surface displacement is related to the magnitude of turbulent velocity in the outer core just above the ICB. Section 4 summarizes the results, and discusses whether Joule heating could be the origin of the elastic anisotropy of the Earth's inner core.

2. Model

We consider an MHD Boussinesq fluid in a sphere. The governing equations determining steady flow and temperature disturbance induced by differential Joule heating are as follows (Takehiro, 2011):

$$0 = -\frac{1}{\rho_0}\nabla p + \alpha T\boldsymbol{g} + \nu\nabla^2\boldsymbol{v}, \tag{1}$$

$$v_r\frac{dT_B}{dr} = \kappa\nabla^2 T + \frac{Q_J}{\rho_0 C_p}, \tag{2}$$

$$\nabla\cdot\boldsymbol{v} = 0. \tag{3}$$

$\boldsymbol{v}$ is velocity, v_r is the radial component of velocity, ρ_0 is the mean density of the Boussinesq fluid, p is pressure, T is the temperature disturbance, and dT_B/dr is the radial temperature gradient of the basic state. Gravity induced by the mass of the sphere itself is a spherically symmetric distribution, $\boldsymbol{g} = -(g_0/a)\boldsymbol{r}$, where g_0 is the gravitational acceleration at the surface, a is the radius of the sphere, and $\boldsymbol{r}$ is the position vector in the radial direction. $Q_J = |\boldsymbol{J}|^2/\sigma = |\nabla\times\boldsymbol{B}|^2/\mu\sigma$ is the Joule heating produced by the magnetic field $\boldsymbol{B}$ diffusing from the outer boundary (ICB) to the interior, where μ and σ are the magnetic permeability and electric conductivity. Note that Eqs. (1) and (2) neglect second order nonlinear terms, the validation of which was discussed in Takehiro (2011).

The difference between these governing equations and those of Takehiro (2011) is the boundary conditions at the ICB, where the effects of surface displacement emerge. The normal stress is balanced at the surface with a buoyancy force proportional to the density difference of the inner and outer core. The temperature at the surface is equal to the melting point, which is varied by the surface displacement. The tangential stresses vanish at the surface:

$$\sigma_{rr} = -p + 2\rho_0\nu\frac{\partial v_r}{\partial r} = -\Delta\rho g h, \tag{4}$$

$$\sigma_{r\theta} = \rho_0\nu\left(\frac{1}{r}\frac{\partial v_r}{\partial\theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right) = 0,$$
$$\sigma_{r\phi} = \rho_0\nu\left(\frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r} + \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}\right) = 0, \tag{5}$$

$$T = \frac{dT_m}{dr}h, \quad \text{at} \quad r = a. \tag{6}$$

Here $\Delta\rho$ is the density difference between the inner and outer core, $h(\theta,\phi)$ is the surface displacement distribution, θ and ϕ are colatitude and azimuth, respectively, and dT_m/dr is the melting temperature gradient. For simplicity, stress and temperature are evaluated at $r = a$, which is the boundary where the surface displacement vanishes.

The non-divergent flow field is expressed with the toroidal and poloidal potentials, ψ and Φ, defined by

$$\boldsymbol{v} = \nabla\times(\psi(r,\theta,\phi)\boldsymbol{r}) + \nabla\times\nabla\times(\Phi(r,\theta,\phi)\boldsymbol{r}), \tag{7}$$

Eqs. (1) and (2) become

$$\nabla^2 L_2\psi = 0, \tag{8}$$

$$\nu\nabla^2 L_2\nabla^2\Phi - \alpha(g_0/a)L_2 T = 0, \tag{9}$$

$$\frac{L_2\Phi}{r}\frac{dT_B}{dr} = \kappa\nabla^2 T + \frac{Q_J}{\rho_0 C_p}. \tag{10}$$

From Eq. (8), $\psi \equiv 0$, meaning that the toroidal component is not induced. Removing the temperature disturbance from Eqs. (9) and (10),

$$\frac{L_2\Phi}{r}\frac{dT_B}{dr} - \frac{\kappa\nu}{\alpha(g_0/a)}\nabla^2\nabla^2\nabla^2\Phi = \frac{Q_J}{\rho_0 C_p}. \tag{11}$$

The boundary conditions are expressed with the velocity potentials. By taking the horizontal divergence of Eq. (1), pressure can be expressed with the potentials. Then, Eqs. (4)–(6) become

$$\rho_0\nu\frac{\partial}{\partial r}r\left(-\nabla^2\Phi + \frac{2L_2\Phi}{r^2}\right) = -\Delta\rho g h \text{ at } r = a, \tag{12}$$

$$\frac{\partial^2\Phi}{\partial r^2} - \frac{2\Phi}{r^2} + \frac{L_2\Phi}{r^2} = 0 \text{ at } r = a, \tag{13}$$

$$\frac{\nu a}{\alpha g_0}\nabla^2\nabla^2\Phi = \frac{dT_m}{dr}h, \text{ at } r = a. \tag{14}$$

Following the procedure of Takehiro (2011), the governing equations are non-dimensionalised, considering the dominance of advection of basic temperature. Using the temperature rising rate $|Q_J|/\rho C_p$ and the difference between basic and adiabatic temperature at the center, ΔT, the time scale is chosen to be $\Delta T\rho C_p/|Q_J|$. The length scale is chosen to be the radius of the sphere a. Then, the poloidal potential should be normalised by $(|Q_J|/\rho C_p)(a^2/\Delta T)$. Eq. (11) becomes

$$\frac{L_2\Phi}{r}\frac{dT_B}{dr} - \frac{1}{R}\nabla_2\nabla_2\nabla_2\Phi_* = q_J, \tag{15}$$

where $q_j = Q_J/|Q_J|$ is non-dimensionalised Joule heating, and R expresses the strength of stable stratification,

$$R = \frac{\alpha g_0\Delta T a^3}{\kappa\nu}. \tag{16}$$

The boundary conditions, Eqs. (12)–(14) are normalised as:

$$\frac{\partial}{\partial r}r\left(-\nabla^2\Phi + \frac{2L_2\Phi}{r^2}\right) = -R_s h, \text{ at } r = 1, \tag{17}$$

$$\frac{\partial^2\Phi}{\partial r^2} - \frac{2\Phi}{r^2} + \frac{L_2\Phi}{r^2} = 0 \text{ at } r = 1, \tag{18}$$

$$\frac{1}{R}\nabla^2\nabla^2\Phi = -\Gamma_m h, \text{ at } r = 1, \tag{19}$$

where

$$\Gamma_m = \frac{(-dT_m/dr)a}{\Delta T} = \frac{(dT_m/dP)\rho g a}{\Delta T}, \quad R_s = \frac{\rho C_p\Delta T}{|Q_J|}\frac{\Delta\rho g a}{\rho_0\nu}. \tag{20}$$

Given the values of R, Γ_m, and R_s, the steady flow and temperature disturbance fields can be obtained from these equations by setting the distributions of basic temperature gradient dT_B/dr, Joule heating q_J, and surface displacement h.

To solve the governing equations with the boundary conditions numerically, the poloidal potential Φ is expanded with spherical harmonic functions in the horizontal directions, and with the polynomials developed by Matsushima and Marcus (1995) in the radial direction. The surface displacement h is also expanded with spherical harmonics. Then, the problem becomes a system of linear equations for each spherical harmonic component of Φ, since the governing equations and boundary conditions are linear. The polynomials for the radial direction are calculated to the 63rd degree.