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Numerical simulations of thermal convection in rotating spherical shells under laboratory conditions

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ABSTRACT

An exhaustive study, based on numerical three-dimensional simulations, of the Boussinesq thermal convection of a fluid confined in a rotating spherical shell is presented. A moderately low Prandtl number fluid ($\sigma = 0.1$) bounded by differentially-heated solid spherical shells is mainly considered. Asymptotic power laws for the mean physical properties of the flows are obtained in the limit of low Rossby number and compared with laboratory experiments and with previous numerical results computed by taking either stress-free boundary conditions or quasi-geostrophic restrictions, and with geodynamo models. Finally, using parameters as close as possible to those of the Earth's outer core, some estimations of the characteristic time and length scales of convection are given.

1. Introduction

The study of thermal convection in rotating spherical geometries is important for understanding the generation of the magnetic fields of cosmic bodies and in particular in the Earth's outer core. It is widely accepted that they are generated by convection of electrically conducting fluids in their interiors. A recent study (Christensen and Aubert, 2006) predicts that the strength of the magnetic fields is independent of the conductivity and of the rotation rate, and that it is basically controlled by the buoyancy flux. Although it is known that the Earth's magnetic field is driven by both thermal and compositional buoyancy forces (Lister and Buffet, 1995; Poirier, 2000) in this paper we focus on the imprint of the former because its knowledge for low Prandtl numbers, σ, is not yet completely understood.

There is a multitude of papers devoted to study the role that thermal convection plays in the dynamics of the celestial bodies. Good reviews can be found in the literature, see for instance Cardin and Olson (1994), Gillet and Jones (2006) and Olson (2011). Therefore we will restrict ourselves to remarking mostly those directly related with temporally chaotic and turbulent flows, with which we compare.

Fully developed thermal convection in a rapidly rotating spherical shell was studied experimentally with water ($\sigma = 7$) and numerically with a quasi-geostrophic model by Cardin and Olson (1994). At a Rayleigh number $Ra = 200Ra_c$, where Ra_c is the critical value of the onset of convection, they found large scale flows with radial variations of the azimuthally averaged velocity, negative (retrograde) near the inner boundary, and positive (prograde), comparable in strength to the former, near the outer. Their results are consistent with a zonal flow maintained by a transfer of energy from the small to the large scales of convection via Reynolds stresses, i.e. by the correlation of the azimuthal component of the velocity field and the cylindrical radial component. This theory was revisited by Gillet and Jones (2006) and Plaut et al. (2008), the latter taking into account the tilt of the convective vortices.

The role of the bulk viscosity and the Ekman friction on the formation of banded zonal structures at weakly nonlinear regimes because of deep convection was studied by Morin and Dormy (2006) with a two-dimensional approximation. They notice that the Ekman pumping tends to produce a large number of bands in the system with the large amplitudes located near the center of the shell. The scale of the transition from dominant bulk viscosity to dominant Ekman friction was found to be $\delta \sim \mathcal{O}(\mathcal{L}E^{1/4})$, where $\mathcal{L}$ is the large scale of the axisymmetric flow. The generation of banded atmospheres in the major planets due to turbulent convection was investigated in Aurnou and Olson (2001) and Aurnou and Heimpel (2004), among others, for different radius ratios and boundary conditions.

In a recent experiment with liquid sodium ($\sigma = 0.01$) Shew and Lathrop (2005) got Ekman numbers as low as 10^{-8}, and reached Rayleigh numbers up to 10^9. They found the experimental law

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for the dimensional time-averaged azimuthal velocity $(|\overline{v_\varphi}|)^d$ as a function of *Ra* and *E* close to the inner boundary of the shell, and also derived the scaling relation $|\overline{v_\varphi}| \sim RaE$ either by balancing Coriolis and buoyancy forces or by supposing that the azimuthal motions come from thermal winds.

The generation of mean zonal flows was analyzed numerically in Christensen (2002) for full three-dimensional calculations with stress-free boundary conditions and Prandtl numbers mainly of order one. It was shown that, at moderate *Ra*, the contribution of the mean zonal flow to the total kinetic energy decreases by increasing σ, but when *Ra* is higher the maximum fraction of the kinetic energy density in the zonal wind is nearly independent of σ. Moreover it was established that the ratio of total kinetic energy density to that in the zonal flow decreases drastically at high *Ra* due to the loss of geostrophy of the convective columns and the corresponding decorrelation of the Reynolds stresses. In agreement with Cardin and Olson (1994) a prograde mean zonal flow close to the outer boundary and retrograde near the inner sphere was found. Asymptotic power laws computed in the limit of negligible viscosity where used successfully to determine the magnitude of the zonal winds on the surface of the giant gas planets in the absence of magnetic field.

Aubert, 2005 and Christensen and Aubert, 2006 also obtained differential rotation with non-slip boundary conditions. In addition, the influence of the magnetic fields on the convective and zonal flows was studied in these papers. In the former, it was found that the magnetic field has no direct influence on the variations of the zonal flow along the axis of rotation, but it allows an enhancement of the heat flux by means of the relaxation of the Taylor–Proudman constraint. However, in the second paper, and in agreement with Soderlund et al. (2012), not much different scalings were obtained for magnetic and non-magnetic (Christensen, 2002) rotating convection. Coherent extrapolations in the presence of magnetic fields were reported in Christensen and Aubert (2006). They predicted a magnetic field strength of order 1 mT inside the Earth's outer core.

The zonal circulations in the Earth's outer core were also investigated experimentally in Aubert et al. (2001), Gillet et al. (2007) and Shew and Lathrop (2005) for water $\sigma = 7$, gallium $\sigma = 0.023$ and liquid sodium $\sigma = 0.010$. In the experiments the gravity force is supplied by the centrifugal acceleration, thus changing the radial dependence of the buoyancy force to normal to the axis of rotation. However, this modification resulted to be of secondary importance for very high *Ra* at low and mid-latitudes. Aurnou (2007) compared heat transfer data from recent models of rotating convection. From the analysis of these data he concluded that numerical and laboratory experiments produced different scaling laws. While for the former the relation between the modified Nusselt number (based on the total heat flux) and the modified heat flux-based Rayleigh number is $Nu^* \sim (Ra^*_{Q_T})^{0.55}$, for the latter it is $Nu^* \sim (Ra^*_{Q_T})^{0.29}$ in the regime dominated by the rotation. According to the author the discrepancy could be due to the low conductivity of the materials employed in the outer spherical shells, which could offer a higher resistance to heat transfer than the fluid, or to the existence of a new scenario in which a regime with the heat transfer controlled by the physical properties of the thermal boundary layers exists, that has not yet been achieved.

General scaling laws for the velocity field depending on the total heat flux-based Rayleigh number, Ra_{Q_T}, E, σ and on the slope of the outer sphere were derived in Cardin and Olson (1994). They approximate the inertial state by neglecting the dissipation through the Ekman boundary layers. The velocity, $\tilde{U} \sim E^{1/5} Ra_{Q_T}^{2/5}$, and length scales, $\delta \sim E^{3/5} Ra_{Q_T}^{1/5}$, for the convective vortices in the high Reynolds number regime were given. In Aubert et al. (2001) scaling laws for the zonal flow, $\hat{U} \sim E^{9/10} Ra_{Q_T}^{4/5}$, were obtained extending the former study by including Ekman friction on the outer boundary. They consider that the kinetic energy is transported from the scale of convective vortices to the large-scale flows through Reynolds stresses, i.e. following a reverse cascade of energy. Then it is dissipated by the zonal flow in the viscous boundary layers. A consequence of this hypothesis is that the energy contained in the zonal flow cannot be significantly higher than the energy contained in the convective vortices. With these scalings they extrapolated values for the Earth's outer core, finding for the radial and azimuthal velocities and for the characteristic length of the convective eddies and zonal flow, $\tilde{U}^d \approx 10^{-3}$ m/s, $\hat{U}^d \approx 10^{-2}$ m/s, $\delta_r = \delta_\varphi \approx 10$ km, respectively.

The same experiment and scaling laws incorporating Ekman pumping and a varying β-effect were re-examined in Gillet et al. (2007), and checked with the data supplied by quasi-geostrophic numerical simulations. At weak supercritical regimes, a quadratic dependence of the zonal flow on convective velocities, $\hat{U} \sim \tilde{U}^2$, was found, reflecting that the zonal flow is driven by the Reynolds stresses if the characteristic length scale corresponds to that established at the onset of convection. For turbulent flows, the amplitude of the zonal motions observed is also larger than that of the convective flows. In this regime they took as characteristic azimuthal scale the Rhines scale, $\delta_\beta \sim \sqrt{\hat{U}/\beta}$, associated with the radial shear of the mean flow. They found that the balance between the Reynolds stress and the Ekman friction is $\hat{U} \sim \tilde{U}^{4/3}$.

Recently, from a dataset of several numerical dynamo models, Stelzer and Jackson (2013) have analyzed, by leaving one parameter out and cross-validation, the results of other authors. They found that diffusion dependent parameters are needed for scaling the flow velocity and the magnetic field strength. Moreover King and Buffett (2013) have found $l \sim E^{1/3}$ and $U \sim C^{1/2} E^{1/3}$ for typical length scales and speeds, respectively. They come from theoretical scaling analysis (balances between rotation, viscosity and buoyancy), which are fitted to several dynamo simulation data. In the latter formula $C = Ra(Nu - 1)\sigma^{-2}$ accounts for the convective power. According to the authors, there are some evidences that geodynamo simulations with magnetic Prandtl numbers tending to zero would lead to a new regime in which large scale convection and Lorentz forces would be more important than viscosity.

The frequency spectra of the temperature observed by Shew and Lathrop (2005) shows an abrupt change in the slope at a high frequency (knee frequency f_c). It was related with the inverse cascade of energy dissipation pointed out by Aubert et al. (2001) and Gillet et al. (2007) in the framework of the two-dimensional turbulence. Shew and Lathrop (2005) suggested that for frequencies lower than f_c the temperature fluctuations are due to an inverse cascade of energy from the scale of convective flows to the largest scales up to the size of the domain.

Despite the power of modern computers full three-dimensional simulations at high Reynolds numbers with an acceptable wall-clock CPU time must be run using parameters that are far from those of the dynamics of the planetary liquid cores, or limit its number to study a particular phenomenon in a very well planed way. In any case nowadays the simulations are far from reaching the Ekman and Rayleigh numbers of the Earth's outer core. In this paper we adopt the first approach, and extend the previous stress-free numerical study of Christensen (2002), by using non-slip boundary conditions, mainly the lower Prandtl number value $\sigma = 0.1$ and the radius ratio of the Earth's outer core $\eta = 0.35$. Our simulations are performed for four Ekman numbers, *E*, the larger being $E = 10^{-4}$, which, although it is far from those needed for real applications, is sufficiently small to be in the asymptotic regime of the onset of convection (see Garcia et al., 2008). In addition the values used allow us to compare with other authors. The