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A loss function approach to group preference aggregation in the AHP

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Abstract

The Analytic Hierarchy Process is a useful method in aggregating group preference. However, judgments are frequently inconsistent, and, in reality, pairwise comparison matrices rarely satisfy the inconsistency criterion. In this situation, we suggest a new method, called a loss function approach, that uses inconsistency ratio as the group evaluation quality. For this method in detail, we introduce Taguchi's loss function. We also develop an evaluation reliability function to derive group weight. Finally, we provide a step-by-step numerical example of a loss function approach.

Keywords: Group decision; Analytic hierarchy process; Inconsistency; Evaluation quality; Loss function

1. Introduction

The essential aspect of multi-criteria decision making is to choose the best alternative from a set of competing alternatives that are evaluated under conflicting criteria. The Analytic Hierarchy Process (AHP) as a multi-criteria decision-making method provides us with a comprehensive framework for solving such decision problems by quantifying the subjective judgments.

The AHP, as a growing field in both its theoretical and applied ramifications, has been applied widely in decision-making problems [1–7]. One of the topics on which research concentrates is the problem of group judgments aggregation and inconsistency ratio. The AHP has been criticized and enhanced on aggregation and inconsistency of individual judgments necessary for group decision making by many researchers, such as Aczel and Saaty [8–17]. However, they have focused mainly on the rules of aggregation, such as geometric mean or arithmetic mean, as well as inconsistency ratio itself. In other words, there were few studies on aggregating group judgments by using inconsistency ratio.

The previous group aggregation methods keep Saaty's rule, which states the inconsistency ratio of individual pairwise comparisons should be less than 0.1. That is, in aggregating individual judgments to a group opinion, one takes only individual judgments with inconsistency ratios less than 0.1. However, practically, pairwise comparison matrices rarely satisfy the inconsistency criterion [18,19], hence these methods tend to ignore much evaluation information by using Saaty's inconsistency ratio.

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Applying the AHP to real world, one of the problems that we should resolve is to satisfy inconsistency test. The original AHP tends to exclude evaluators' pairwise comparison matrices for that their inconsistency ratios are over 0.1. Thus, the loss of much information on evaluators' judgments would occur. Kapapetrovic and Rosenbloom [18] pointed in their paper as follows: "the pairwise comparison matrices are frequently inconsistent and the decision makers are quite conscientious in evaluating the pairwise comparisons. The pairwise comparisons the decision makers make are rarely random even if they fail the consistency test."

In order to overcome such problems, we introduce the concept of Taguchi's loss function [20,21] and develop a loss function approach, which is a new method for improving group judgments aggregation. Thus, all pairwise comparison matrices can be used by this approach. In order to do, this study is organized five sections as follows. Section 2 reviews the literature related to group decision making and inconsistency ratio. Section 3 deals with the design of a loss function approach. Section 4 provides a step-by-step numerical example as a convincing case with which we compared the original AHP with a loss function approach. Lastly, Section 5 says conclusion and limitations of this study.

2. Literature review

2.1. Group decision making in the AHP

Group decision making involves weighted aggregation of different individual preferences to obtain a single collective preference. This subject has received a great deal of attention from researchers in many disciplines. Unfortunately, it is extremely difficult to accurately assess and quantify changing preferences and to aggregate conflicting opinions held by diverse group.

Azcel and Saaty [8,9] proposed a functional equation approach to aggregate the ratio judgments. Let us suppose that the numerical judgments $x_1, x_2, \ldots, x_m$ given by m persons lie in a continuum (interval) P of positive numbers so that P may contain $x_1, x_2, \ldots, x_m$ as well as their powers, reciprocals and geometric means, etc. The aggregating function will map P_m into a proper interval J, and $f(x_1, x_2, \ldots, x_m)$ will be called the result of the aggregation for the judgments $x_1, x_2, \ldots, x_m$. The function should satisfy the separability condition, unanimity condition and reciprocal condition. It is the geometric mean as the following equation:

$$f(x_1, x_2, \ldots, x_m) = (x_1, x_2, \ldots, x_m)^{1/m}. \tag{1}$$

In addition to the geometric mean method discussed above, the arithmetic mean may also be used to aggregate group judgments. The only difference is that the arithmetic mean can only be applied to the final priority weights. This is because of the reciprocal property of pairwise comparison: $1/\sum_{i=1}^{m} a_{jki} \neq \sum_{i=1}^{m} 1/a_{jki}$. The arithmetic mean method cannot be used to aggregate the pairwise comparison matrix A. So, we have the mathematical form of the arithmetic mean operating on the priority weight as follows:

$$V_i = f(A_i) = (\{v_1\}_i, \ldots, \{v_n\}_i), \quad i = 1, 2, \ldots, m, \tag{2}$$

$$\bar{V} = (\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_n) = \left(\frac{1}{m} \sum_{i=1}^{m} \{v_1\}_i, \frac{1}{m} \sum_{i=1}^{m} \{v_2\}_i, \ldots, \frac{1}{m} \sum_{i=1}^{m} \{v_n\}_i \right), \tag{3}$$

where v is the final priority weight.

Ramanathan and Ganesh [22] showed that geometric mean method (GMM), when employed in AHP, is not suitable for group decision making with well-established social choice axioms, which govern the process of combining individual opinions to obtain a single group opinion. They explained the justification of the weighted arithmetic mean method (WAMM) through the social choice axioms. But, in order to use the WAMM, one has to find the weights to be assigned to the members of the group. This is often a difficult task, especially if the group is large, as in the case of public policy decisions and when judgments are elicited through the use of questionnaires.

Forman and Peniwati [12] discussed two different methods for synthesis of individual judgments. They claimed that the choice of method depends on whether the group acts together (aggregating individual judgments as a unit, AIJ) or as separate individuals (aggregating individual priorities, AIP). Their example of a group acting together as a unit was a group of department heads meeting to decide on corporate policy. Their example of a group acting as separate individuals was representative constituencies such as taxpayers with stakes in welfare reform. They stated that both the