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Magnetic fields generated by hydromagnetic dynamos at the low Prandtl number in dependence on the Ekman and magnetic Prandtl numbers

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ABSTRACT

This article investigates the dependence of hydromagnetic dynamos on the magnetic Prandtl number at low Prandtl number. In all the investigated cases, the generated magnetic fields are dipolar and neither transition to hemispherical dynamos nor weaker magnetic fields (which are less dipole dominated) were observed, although the inertia becomes important. The magnetic field becomes weak in the polar regions (is "convected out of polar regions") only for low Prandtl numbers, when the inertia becomes important. It is a basic condition. However, whether the magnetic field gets weak in the polar regions (is "convected out of polar regions") or not depends also on the magnetic Prandtl number. The magnetic Prandtl number has to exceed a minimum value in order to sustain dynamo action. If the magnetic diffusion is small (large magnetic Prandtl numbers) then this phenomenon does not exist but if it is large (small magnetic Prandtl numbers) it exists because the strong magnetic diffusion significantly weakens the magnetic field inside the tangent cylinder. The magnetic diffusion and inertia seem to act in the same direction as to weaken the magnetic field inside the tangent cylinder.

1. Introduction

Cosmic magnetohydrodynamics and the theory of planetary and solar dynamos provide the best explanation of magnetic field generation mechanism in various objects in the universe. They are able to describe an origin, spatial and temporal evolution of cosmic magnetic fields and conditions, which must be satisfied for the dynamo action (Roberts and Glatzmaier, 2000; Glatzmaier, 2005; Christensen and Wicht, 2007; Kaiser, 2009). Planetary magnetic fields (also the geomagnetic field) are generated by hydromagnetic dynamos working in their fluid interiors (Roberts and Glatzmaier, 2000; Olson and Glatzmaier, 2005; Christensen and Wicht, 2007). Complicated processes going on in the Earth's and planetary fluid interiors, e.g., a chemical homogenisation, gravitational differentiation, solidification processes acting on the inner core boundary, constitute the driving mechanism of dynamos, i.e. they are the fundamental source of convection or magnetoconvection (Jones, 2000). Numerical modelling of self-consistent dynamos has made noticeable progress in the last 16 years due to the progress in computer technology (e.g., Glatzmaier, 2005; Christensen and Wicht, 2007; Kageyama et al., 2008; Takahashi et al., 2008; Sakuraba and Roberts, 2009; Christensen, 2011; Wicht and Tilgner, 2010; Takahashi and Shimizu, 2012). The results of numerical simulations are in very good agreement with the observations of the recent geomagnetic field and with paleomagnetic research (Roberts and Glatzmaier, 2000; Christensen and Wicht, 2007). The superadiabatic radial temperature gradient between the core-mantle boundary (CMB) and the inner core boundary (ICB) constitutes the main driving force of convection in most dynamo models. The current state of numerical dynamo modelling is described very well in Christensen and Wicht (2007), Kageyama et al. (2008), Takahashi et al. (2008), Sakuraba and Roberts (2009), Christensen (2011), Wicht and Tilgner (2010). Although the numerical results agree with observations, numerical simulations of the geomagnetic field are not able to run in the Earth-like parameter regime because of the considerable spatial resolution that is required (Glatzmaier, 2005; Christensen and Wicht, 2007; Sakuraba and Roberts, 2009; Christensen et al., 2010). Geodynamo models in the Earth-like parameter regime are still a great challenge (Glatzmaier, 2005; Sakuraba and Roberts, 2009; Christensen et al., 2010). The Prandtl number is the only parameter whose geophysical value can be directly used in dynamo models.

In Šimkanin and Hejda (2011) we investigated the simultaneous influence of the non-uniform stratification and viscous, thermal and magnetic diffusive processes on the dynamo action. For the outer Earth's core it is expected that the kinematic viscosity $\nu = 10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}$, the thermal diffusivity $\kappa = 5 \times 10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}$ and the magnetic diffusivity $\eta = 2\,\mathrm{m}^2\,\mathrm{s}^{-1}$ (Fearn, 2007). In most numerical simulations scientists have used $\nu/\kappa = 1$ (the ratio ν/κ is known as the Prandtl number) but for the Earth's core $\nu/\kappa = 0.2$ (Fearn, 2007). Our results showed that the influence of

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non-uniform stratification (it means the density stratification, when the upper part of the shell close to the CMB is stably stratified and the lower part towards the ICB unstably stratified, which is based on the idea of "a stably stratified ocean" at the top of the outer Earth's core) was weak for our parameters but the influence of the Prandtl number was strong. In addition we showed that the ratio ν/η (known as the magnetic Prandtl number) could govern the influence of inertia on dynamo at low Prandtl numbers (Šimkanin and Hejda, 2011). In this paper, we test this hypothesis. The model and governing equations are presented in Section 2 and the parameters used in previous analyses in Section 3. Numerical results are provided in Section 4. Section 5 is devoted to conclusions.

2. Governing equations and model

Here we consider dynamo action due to thermal convection of an electrically conducting incompressible fluid in the Boussinesq approximation in an unstably stratified spherical shell ($r_i < r < r_o$) rotating with angular velocity Ω. The evolution of the magnetic field $\mathbf{B}$, the velocity $\mathbf{V}$ and the temperature T is described by the following system of dimensionless equations:

$$E\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} - \nabla^2\mathbf{V}\right) + 2\mathbf{1}_z \times \mathbf{V} + \nabla P = R_a \frac{\mathbf{r}}{r_o} T + \frac{1}{P_m}(\nabla\times\mathbf{B})\times\mathbf{B}, \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{V}\times\mathbf{B}) + \frac{1}{P_m}\nabla^2\mathbf{B}, \quad (2)$$

$$\frac{\partial T}{\partial t} + (\mathbf{V}\cdot\nabla)T = \frac{1}{P_r}\nabla^2 T, \quad (3)$$

$$\nabla\cdot\mathbf{V} = 0,\ \nabla\cdot\mathbf{B} = 0. \quad (4)$$

The radius of the outer sphere L, is the typical length scale, which makes the dimensionless outer core radius $r_o = 1$; the inner core radius r_i is, similar to that of the Earth, equal to 0.35. (r, θ, φ) is the spherical system of coordinates, $\mathbf{1}_z$ is the unit vector. The time, t, is measured in the unit of L^2/ν, velocity, $\mathbf{V}$, in ν/L, magnetic induction, $\mathbf{B}$, in $(\rho\mu\eta\Omega)^{1/2}$, temperature, T, in ΔT, and pressure, P, in $\rho\nu^2/L^2$. The dimensionless parameters appearing in Eqs. (1)–(4) are the Prandtl number, $P_r = \nu/\kappa$, the magnetic Prandtl number, $P_m = \nu/\eta$, the Ekman number, $E = \nu/\Omega L^2$ and the modified Rayleigh number $R_a = \alpha g_0 \Delta T L/\nu\Omega$, where κ is the thermal diffusivity, ν is the kinematic viscosity, μ is the magnetic permeability, η is the magnetic diffusivity, ρ is the density, α is the coefficient of thermal expansion, ΔT is the drop of temperature through the shell and g_0 is the gravity acceleration at $r = r_o$.

Eqs. (1)–(4) are closed by the non-penetrating and no-slip boundary conditions for the velocity field at the rigid surfaces and fixed temperature boundary conditions (the constant temperature $T_i = 1$ and $T_o = 0$ at the inner and outer boundaries of the shell, respectively). The outer boundary is electrically insulating (the magnetic field on this boundary matches with the appropriate potential field in the exterior which implies no external sources of the field), while the inner boundary is electrically conducting (electrical conductivity of the outer and inner core is considered to be the same).

3. Parameters in previous analyses

Viscous, thermal and magnetic diffusive processes significantly influence the dynamo. In most numerical simulations $P_r = 1$ is generally used, but for the outer Earth's core the Prandtl number $P_r = 0.2$ is expected (Fearn, 2007). It is necessary to remark that some authors indicate $P_r = 0.1 - 1$ (see, e.g., Christensen and Wicht, 2007). Let us summarize the results of previous analyses and our results related to the Prandtl and magnetic Prandtl numbers. Christensen et al. (1999) and Christensen and Aubert (2006) showed that the minimal value of the magnetic Prandtl number at which dipolar dynamos exist varies with the Ekman number as

$$P_{m_{min}} \simeq 450E^{3/4}. \quad (5)$$

This relation was confirmed at $P_r = 1$ and $E \geqslant 3\times10^{-6}$. For $P_r < 1$ or $P_r > 1$ Christensen and Aubert, 2006 always used $P_m \geqslant 1$. However, Busse and Simitev (2005) found dipolar dynamos for $P_r < 1$ (they used $P_r = 0.1$) and $P_m \sim 0.15$, which is slightly greater than $P_{m_{min}}$. Sreenivasan and Jones (2006a) found dipolar dynamos at $P_r = P_m = 0.5$. Consequently, the relation (5) seems to be valid also for $P_r < 1$ and $P_m < 1$.

For $P_r \geqslant 1$ and $P_m \geqslant 1$ dynamos are mostly dipolar, large-scale flows are columnar and magnetic fields are never convected out of polar regions. These cases are the most investigated ones and they are presented, e.g., in Olson et al. (1999), Christensen et al. (1999), Christensen and Aubert (2006), Glatzmaier (2005), Christensen and Wicht (2007), Sreenivasan and Jones (2006a) (Sreenivasan and Jones, 2006a at $P_m = P_r = 5$ and 1). For $P_r < 1$ the inertia becomes important but its influence on the dynamo depends on the value of P_m (Busse and Simitev, 2005; Busse and Simitev, 2011; Sreenivasan and Jones, 2006a; Šimkanin and Hejda, 2011). The case $P_r < 1$ and $P_m \geqslant 1$ (rather $P_m \gg P_{m_{min}}$) is similar to the previous case $P_r \geqslant 1$ and $P_m \geqslant 1$. Dynamos are mostly dipolar, large-scale flows are columnar and magnetic fields are never convected out of polar regions although the inertia would be important in this case. However, for $P_r < 1$ and $P_m < 1$ (rather $P_m \leqslant P_{m_{min}}$) the inertia significantly influences the dynamo and it is possible to observe the breakdown of the columnar structure of the convection because the dipolar structure breaks generally down (the magnetic field weakens considerably). As fluid motion becomes strong in the polar regions, the magnetic field is convected out of polar regions (Sreenivasan and Jones, 2006a). Busse and Simitev (2005) used $P_r = 0.1$ in their simulations and they observed at low values of P_m a transition to hemispherical dynamos and at even lower values of P_m a further transition to quadrupolar dynamos. These transitions were observed neither in Sreenivasan and Jones (2006a) nor in Šimkanin and Hejda (2011) but it is necessary to remark that they used stress-free boundary conditions, while Sreenivasan and Jones (2006a) and Šimkanin and Hejda (2011) used no-slip boundary conditions. However, Hori et al. (2010) observed the transition to hemispherical and quadrupolar dynamos even with no-slip boundary conditions and this difference was caused by thermal boundary conditions. In addition, in Hori et al. (2010) and Busse and Simitev (2005) the convection was driven by the internal heating, while in Sreenivasan and Jones (2006a) and Šimkanin and Hejda (2011) by the temperature difference between outer and inner boundaries. Sreenivasan and Jones (2006a) used $P_r = 0.2$ and 0.5. At $P_m = P_r = 0.5$ they observed the dipolar dynamo, the columnar large-scale flow and the magnetic field, which was not convected out of polar regions, while at $P_m = P_r = 0.2$ they observed the non-dipolar dynamo, the breakdown of the columnar structure of the convection in consequence of the dipolar structure breaks generally down, and the magnetic field, which was convected out of polar regions due to strong fluid motion in the these regions.

We decided to use in our numerical simulations both $P_r = 0.2$ and also 1 in order to compare these two cases. The simultaneous influence of non-uniform stratification and diffusive processes on the dynamo action was investigated in Šimkanin and Hejda (2011). We considered a model, in which 10% of the shell is stably stratified (the upper sub-shell is stably stratified) and 90% unstably