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Hydromagnetic quasi-geostrophic modes in rapidly rotating planetary cores

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ABSTRACT

The core of a terrestrial-type planet consists of a spherical shell of rapidly rotating, electrically conducting, fluid. Such a body supports two distinct classes of quasi-geostrophic (QG) eigenmodes: fast, primarily hydrodynamic, inertial modes with period related to the rotation time scale and slow, primarily magnetic, magnetostrophic modes with much longer periods. Here, we investigate the properties of these hydromagnetic quasi-geostrophic modes as a function of non-dimensional parameters controlling the strength of the background magnetic field, the planetary rotation rate, and the amount of magnetic dissipation.

We first present analytic solutions that illustrate the essential parameter dependences of the modes and provide a convenient benchmark for our numerical scheme. A comparison between known three-dimensional inertial modes in a sphere and our axially invariant QG modes shows encouraging agreement at low azimuthal wavenumbers, particularly for the slowest modes. The container geometry and background magnetic field structure are found to influence the radial structure of the modes, but not the scaling of their frequency with the control parameters. When the background magnetic field decreases toward the outer boundary in a spherical shell, QG modes tend to be compressed towards the outer boundary. Including magnetic dissipation, we find a continuous transition from diffusionless slow magnetic modes into quasi-free decay magnetic modes. During that transition (which is controlled by the magnitude of the Elsasser number), we find that slow magnetic modes weakly modified by diffusion exhibit a distinctive spiralling planform. When magnetic diffusion is significant (Elsasser number much smaller than unity), we find quasi-free decay slow magnetic modes whose decay time scale is comparable to, or shorter than, their oscillation time scale.

Based on our analysis, we expect Mercury to be in a regime where the slow magnetic modes are of quasi-free decay type. Earth and possibly Ganymede, with their larger Elsasser numbers, may possess slow modes that are in the transition regime of weak diffusion, depending on the details of their poorly known internal magnetic fields. Fast QG modes, that are almost unaffected by the background magnetic field, are expected in the cores of all three bodies.

1. Introduction

Electrically conducting fluids permeated by magnetic fields support hydromagnetic waves (Alfvén, 1942). These waves play an important dynamical role. They are a means of transporting energy and momentum, and are the natural response of the system to forcing. The cores of terrestrial-type planets, consisting of approximately spherical shells of rapidly-rotating liquid metal, will support such waves provided they also possess an intrinsic magnetic field. Hydromagnetic waves in planetary cores are strongly influenced by rapid rotation which leads to the existence of two classes of waves, one with a time scale comparable to that of the rotation and another class with time scale much longer than that of rotation (Lehnert, 1954; Braginsky, 1964; Hide, 1966). The latter has been proposed as a candidate for producing geomagnetic secular variation and may also contribute to the time variations of other planetary magnetic fields. Since planetary cores are closed containers, it is more appropriate to discuss free oscillations (or eigenmodes) rather than progressive waves in an unconfined system; we therefore use the terminology hydromagnetic modes hereinafter.

In this study we determine the hydromagnetic modes for a simple quasi-geostrophic (QG) model of planetary core dynamics, building on the pioneering studies of Hide (1966), Malkus (1967) and Busse (1976). Although hydromagnetic modes are often cited as an important ingredient of planetary core dynamics, there have

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been surprisingly few studies of them following these early important investigations. The QG model (Busse, 1970; Pais and Jault, 2008; Canet et al., 2009) assumes that disturbances are to leading order invariant parallel to the rotation axis. Studies of three-dimensional instabilities and waves in rapidly-rotating hydromagnetic systems have demonstrated that the QG model is a useful approximation provided the period of the disturbance considered is slow compared to the rotation period (Zhang and Fearn, 1994; Gillet et al., 2011) but faster than the magnetic diffusion time scale (Schmitt, 2012).

Our model includes the spherical shell geometry essential for studies of planetary cores, rather than relying on a local β-plane (Hide, 1966) or annulus (Busse, 1976) approximation or being restricted to a full sphere (Malkus, 1967). Besides assuming it to be uniform (Hide, 1966) or linearly increasing with radius (Malkus, 1967), we also consider the possibility of an underlying azimuthal magnetic field that reaches a maximum value within the shell, in line with the profile found by Gillet et al. (2010) for the cylindrical radial magnetic field. Furthermore, we include realistic levels of magnetic dissipation in the plane perpendicular to the rotation axis where the smallest length scales of the field perturbation are likely to be located. Global mode solutions rather than local solutions are obtained numerically. The investigations presented here are complementary to recent three-dimensional investigations of hydromagnetic modes by Schmitt (2010, 2012) that focused on so-called quasi-free decay modes, which are closely related to the magnetic decay modes of a stationary conductor. In order to concentrate on the intrinsic mode properties, we do not explicitly consider the forcing of the hydromagnetic modes; they may be forced, for example, by convection (e.g. Fearn and Proctor, 1983; Zhang, 1995), shear, or magnetic field instability (e.g. Acheson, 1972; Zhang and Fearn, 1994). Our primary motivation is to better understand the linear dynamics possible within more complete (forced, nonlinear) QG models of rapid core dynamics, currently under development with the aim of modelling geomagnetic secular variation (e.g. Canet et al., 2009).

Laboratory experiments involving rapidly rotating liquid metals permeated by strong magnetic fields also provide some evidence for the existence of hydromagnetic modes (Schmitt et al., 2008; Nornberg et al., 2010). The detailed interpretation of these experiments is still under discussion, but it seems that the observed wave-like disturbances are influenced both by the magnetic field and by the rotation of the fluid. There have also been reports that hydromagnetic waves may be responsible for some azimuthal motions of flux patches in numerical geodynamo models (see e.g. Kono and Roberts, 2002). However, both in laboratory experiments and in numerical geodynamo models it has proven difficult to single out the particular wave mechanism at work. Here, by studying a simpler (linear, unforced) problem, and by making the additional assumption that the disturbances are axially invariant, we are able to fully characterize the nature of the oscillations.

Two fundamental non-dimensional parameters are found to control the properties of the hydromagnetic modes in our QG model. The first is the ratio of the rotation time scale over the time scale for Alfvén waves to cross the system, known as the Lehnert number,

$$\mathrm{Le} = \frac{B^\star}{\Omega\sqrt{\rho\mu_0}r_o}. \tag{1}$$

The second is the ratio between the magnetic diffusion time scale and the time scale of Alfvén waves, known as the Lundquist number

$$\mathrm{Lu} = \frac{r_o B^\star}{\eta\sqrt{\rho\mu_0}}. \tag{2}$$

Here the outer core radius r_o has been taken as the length scale and $B^\star$ is the typical magnetic field intensity in the planetary core interior. ρ is the density of the fluid, Ω is the angular rotation rate, μ_0 is the magnetic permeability of free space, and $\eta = 1/\sigma\mu_0$ is the magnetic diffusivity (σ is the electrical conductivity). Table 1 summarizes estimated ranges of the Lehnert and Lundquist numbers for Mercury, Earth and Ganymede. For Earth's core an upper estimate of the field strength comes from the recent study by Gillet et al. (2010) while for Mercury and Ganymede we use an upper estimate of ten times the observed field value downward continued to the core-mantle boundary (as suggested by the ratio between the mean field strength inside the shell and the surface poloidal field strength typically found in numerical geodynamo models).

We focus our discussion on the terrestrial-type planets since their interior most likely includes a spherical shell of liquid metal surrounded by a solid silicate layer, and because, as assumed in our model, incompressibility of the liquid layer is likely to be a reasonable approximation. Since Venus and Mars do not presently possess a strong intrinisic field we discuss only Mercury, Earth and Jupiter's largest moon Ganymede, which also has a dynamo generated internal field. Due to the lack of information concerning the aspect ratio of the inner core to outer core radii in Ganymede and Mercury, only the influence of changing Le and Lu (both of which depend on r_o) is studied here, although the aspect ratio will certainly affect both the period and spatial structure of the eigenmodes. Both the spatial and temporal spectrum of magnetic variations on Earth are now well known (Hulot et al., 2010). The nature of secular variation is currently unknown on Mercury (Anderson et al., 2008, 2012) and on Ganymede (Kivelson et al., 2002). Ambitious orbital missions carrying magnetometers are now planned to survey Mercury [ESA's Bepi Colombo mission - see Yamakawa et al., 2004] and Ganymede [ESA's Jupiter Icy Moon Explorer, JUICE – see Dougherty et al., 2012] so it is of some interest to consider the role hydromagnetic QG modes could play in the magnetic secular variation of these bodies. Table 1 shows that the Lehnert number is estimated to be very small for the cores of Mercury, Earth, and Ganymede, implying that rotation dominates magnetic forces for transient motions, suggesting that a QG model is appropriate (Jault, 2008) for studying them. Estimates of the Lundquist number vary by several orders of magnitude, due to uncertainties both in the field strength and in the electrical resistivity of the planetary cores. Nonetheless, it is certainly largest for the Earth and smallest for Mercury. We will show that the Lunquist number plays a crucial role in determining the relevance of slow magnetic QG modes in planetary core dynamics.

This paper is divided into five sections. Section 2 presents the mathematical formulation of our QG model. Section 3 presents an analytic solution describing many essential details of the modes including the splitting into fast and slow modes and the parameter dependence of the dispersion relations. Detailed numerical results are presented in Section 4. These include the influence of spherical geometry, the influence of the background zonal field structure, and the effects of magnetic dissipation on the modes. In Section 5 the implications of our results for planetary core motions and variations in their magnetic fields are discussed. An important comparison of mode periods in the absence of a magnetic field with the analytic full sphere inertial mode solutions is reported in Appendix A.

2. Model and methods

2.1. Fluid flow

As illustrated in Fig. 1, we consider the dynamics of an incompressible fluid contained in a spherical shell of inner and outer radii