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Numerical investigation of a layered temperature-dependent viscosity convection in comparison to convection with a full temperature dependence

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ABSTRACT

The viscosity of the Earth's mantle is strongly variable. In particular, the dependence on temperature leads to viscosity variations of several orders of magnitude. This is crucial for the modelling of stiff surface plates in mantle convection codes but is a limiting factor in numerical experiments. Therefore, various approximations to reduce the strong vertical gradients have been applied. Here, we present an approximation that also neglects lateral variations. This layered temperature dependence has the advantage that it can easily be implemented into mantle convection codes that cannot handle lateral variations in the viscosity. Furthermore we find that typically convergence rates are improved compared to models using the full temperature dependence so that computation time can be reduced in models that would allow for the full temperature dependence. In this study we compare the results of the horizontally-averaged and the full temperature-dependent viscosity convection for a wide range of parameters comprising all three flow regimes known in thermoviscous convection and for models featuring an additional stress-dependent viscosity to allow for plate motion. Additionally, we discuss why the approximation shows minor differences to the full temperature dependence in some cases and present improvements. In general, we observe that the layered temperature-dependent viscosity convection is a suitable approximation to the full temperature depencence. Fast models of one-plate planets can be run when only using the layered temperature-dependent viscosity and plate-like motion results with an additional stress dependence.

1. Introduction

Mantle viscosity strongly depends on stress, pressure and particular temperature (Kirby, 1983; Karato and Wu, 1993). The effect of a temperature-dependent viscosity on the style of convection has been shown in a number of laboratory, numerical and theoretical studies (e.g. Booker, 1976; Nataf and Richter, 1982; Stengel et al., 1982; Christensen, 1984; Morris and Canright, 1984; Davaille and Jaupart, 1993; Tackley, 1993; Solomatov, 1995; Moresi and Solomatov, 1995, and many more). The general observation in thermoviscous convection is that the surface more strongly decouples from the interior as the strength of the temperature dependence increases, i.e. with a stronger viscosity contrast between the top and bottom boundary. This is a first step towards modelling plates as stiff entities atop the convecting mantle.

Three flow regimes have been identified for different viscosity contrasts. Solomatov (1995) gives the approximate values of the transitions between the flow regimes. For a small viscosity contrast (<100) the surface behaves fluid-like such as the interior. Therefore the small-viscosity contrast regime resembles isoviscous convection. Increasing the viscosity contrast beyond 100 reduces the surface velocity and the surface becomes sluggish compared to the interior. Finally, for viscosity contrasts larger than about 3000 the surface becomes immobile. Therefore this regime is commonly referred to as stagnant-lid convection.

Hansen and Yuen (1993) have additionally found that there is a Rayleigh number-dependent boundary at intermediate viscosity contrasts. At high Rayleigh numbers the surface is again mobilised and convection resembles isoviscous convection as in the small-viscosity contrast regime.

The planform of convection and consequently the depth profiles resulting in each regime differ (Trompert and Hansen, 1998; Schubert et al., 2001). In the small-viscosity contrast regime the horizontal wavelength of the flow is comparable to the fluid layer depth. The flow structure shows that the up- and downflows are in balance and depth profiles are symmetric. With an increasing viscosity contrast a stronger imbalance between the up- and downflows occurs which is also reflected in unsymmetric depth profiles. The planform of convection changes to larger horizontal

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scales in the sluggish-lid regime until the wavelength becomes very small for the stagnant-lid regime (Schubert et al., 2001).

The temperature profiles in the stagnant-lid regime show that across the cold thermal boundary layer almost the entire temperature difference ΔT occurs and that the heat transfer here is largely conductive (Solomatov, 1995). Beneath the stagnant lid, convection is almost isoviscous with only a small rheological temperature difference driving the convection and little cold material sinking into the interior (e.g. Morris and Canright, 1984). As a consequence, the efficiency for heat transfer over the surface is reduced in the stagnant-lid regime and the nondimensional interior temperature is higher than the 0.5 observed in isoviscous convection. Scalings of the heat flow vs. the Rayleigh number show that the exponent in the power-law relation decreases with an increasing viscosity contrast (e.g. Christensen, 1984) until it reaches an asymptotic value in the stagnant-lid regime (Moresi and Solomatov, 1995).

The Rayleigh number in thermoviscous convection is not unique but varies with the viscosity which is given by an Arrhenius law (e.g. Karato and Wu, 1993). In numerical studies an approximation in the form of a linearised exponential function (Frank-Kamenetskii rheology) is commonly applied to limit the strong vertical viscosity gradients occurring in the top boundary layer (e.g. Christensen, 1984; Solomatov, 1995; King, 2009; Stein and Hansen, 2013). Similarly, strong lateral viscosity variations can occur between the cold, highly viscous downflows and the warm, low viscous ambient material.

Therefore, we here present model calculations featuring a layered temperature-dependent viscosity convection, i.e. we use the radial temperature profile to compute the viscosity. Our work is a follow-up study of Tackley (1996) and Sunder-Plassmann and Christensen (2000). Looking at the flow pattern of two 3D simulations, Tackley (1996) finds no qualitative differences in simulations featuring the full temperature dependence and those in which the viscosity is averaged horizontally. A more quantitative 2D study of layered temperature-dependent viscosity convection is provided by Sunder-Plassmann and Christensen (2000). These authors also find that neglecting lateral variations has very little effect. However, their simulations using free-slip boundary conditions have fairly low viscosity contrasts which do not allow for the formation of a stagnant lid. The statement that the layered temperature-dependent viscosity is a good approximation in the stagnant-lid regime relies on simulations with a likewise low viscosity contrast but no-slip conditions. In the present work we extend the 2D and 3D analysis of layered temperature-dependent viscosity convection with free-slip boundary conditions to higher viscosity contrasts mimicking stagnant-lid convection. The regime transitions we obtain are in good agreement with the results obtained for the full temperature dependence as are the scaling laws in all three flow regimes. Adding a deformation mechanism to the system shows that also plate behaviour can be modelled when using the approximation. We discuss limitations and provide applications of the approximation to currently used models.

2. Models

Assuming incompressibility and the Boussinesq approximation the governing nondimensional equations for mantle convection are:

$$\vec{\nabla} \cdot \vec{u} = 0 \tag{1}$$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T - \vec{\nabla}^2 T = 0 \tag{2}$$

$$-\vec{\nabla} p + \vec{\nabla} \cdot \left[\eta\left(\vec{\nabla}\vec{u} + (\vec{\nabla}\vec{u})^T\right)\right] + RaT\vec{e}_z = 0. \tag{3}$$

Here, $\vec{u}$, T, p, η and $\vec{e}_z$ are the velocity vector, the temperature, the pressure, the viscosity and the unit vector in the vertical direction z, respectively. The Rayleigh number is given as:

$$Ra = \frac{\alpha \rho g \Delta T d^3}{\kappa \eta} \tag{4}$$

where α, ρ, g, ΔT, d and κ are the thermal expansion coefficient, the density, the gravitational acceleration, the temperature difference across the fluid layer, the thickness of the fluid layer and the thermal diffusivity, respectively. In thermoviscous convection the Rayleigh number is non-unique due to the variable viscosity. In this study the Rayleigh number is evaluated using the viscosity at the bottom, which is a-priori known.

The viscosity of mantle material is assumed to follow an Arrhenius-typed law that is commonly used in the linearized version to reduce the strong vertical gradients (cf. Schubert et al., 2001; Stein and Hansen, 2013):

$$\eta_T = \exp\left(-\ln(\Delta\eta)T\right) \tag{5}$$

Here, $\Delta\eta = \eta_0/\eta_1$ is the viscosity ratio between the top and the bottom boundary. In the layered temperature-dependent viscosity convection only the horizontally-averaged temperature $< T >$ is applied to further eliminate the horizontal viscosity variations:

$$\eta_{<T>} = \exp\left(-\ln(\Delta\eta) < T >\right). \tag{6}$$

$< T >$ is computed as the arithmetic mean for most of this study. Only in Section 3.3 we do also apply the midrange method.

We use two numerical models. In the first model, MC3D, the temperature equation is solved using an explicit finite difference method and for the velocities the spectral formulation is used (Gable et al., 1991). The code cannot easily handle lateral viscosity variations because in the spectral formulation variables are formulated as a sum of global basis functions (e.g. Fourier expansion in Cartesian models with periodic boundary conditions or Chebyshev/Legendre polynomials in the case of non-periodic boundaries and spherical harmonics in spherical calculations) instead of local functions like in the spatial methods. For lateral viscosity variations the momentum equation is non-linear so that the modes do not decouple and cannot be solved independently as in isoviscous convection. An easy way to preserve the efficiency of the spectral method is to approximate the temperature dependence by using a temperature profile.

For the comparison of the full temperature dependence and the approximation we use a model that can handle both viscosities. The second code uses an implicit, finite volume multigrid method to solve the equations of mantle convection (cf. Trompert and Hansen, 1996). Horizontal boundary conditions for all simulations are set to $T = 0$ (top) and $T = 1$ (bottom) and a vanishing heat flux for the temperature is assumed at the vertical boundaries. For the velocities all boundaries are free-slip. Square boxes with a resolution of 64^2 (64^3) control volumes are used. Additionally a grid refinement is applied to better resolve the strong gradients appearing in the thermal boundary layers.

3. Numerical experiments

Using the layered temperature-dependent viscosity function (Eq. 6) in the spectral code we are able to get a rigid surface without prescribing a certain viscosity profile as was done in previous studies employing MC3D (e.g. Stein and Lowman, 2010; Lowman et al., 2011). Fig. 1 shows that with an increasing viscosity contrast the different flow regimes as described by Solomatov (1995) are obtained. The surface Nusselt number decreases with increasing viscosity contrast (Fig. 1a) as the top layer grows thicker and becomes highly viscous compared to the convecting interior