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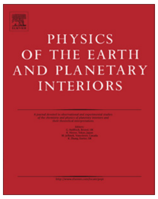

# Numerical modeling of deformation and stress fields around a magma chamber: Constraints on failure conditions and rheology

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## ABSTRACT

We present a stress–strain analysis using the Finite Element Method to investigate failure conditions of pressured magma chambers embedded in an inelastic domain. The pressure build-up induces variations in the stress field until failure conditions are reached. Therefore, the definition of the failure conditions could have a significant impact on the volcano hazard assessment. Using a numerical approach, we analyze the stresses in a gravitationally loaded model assuming a brittle failure criterion, to determine the favorable conditions for magma chamber failure in different source geometries, reference stress states, pore fluid pressures, rock rheologies and topographic profiles. The numerical results allow us to pinpoint the conditions promoting seismicity near the magma chamber. The methodology places a limit on the pressure that a magma chamber can sustain before failing and provides a quantitative estimate of the uplift expected at the ground surface. Thermally-activated ductile regimes, which may develop in the region surrounding a heated magma chamber, are also investigated. The stress relaxation in a ductile shell may prevent the wall rupture, favoring the growth of large overpressured chambers, which could lead to considerable deformation at the ground surface without significant seismicity. The numerical results suggest that a spherical source, compressive regime, gentle edifice topography, and growth of a ductile shell are important factors for the initial formation and the mechanical stability of magma storage systems. On the other hand, an elongated ellipsoidal source, extensional regime, steep volcano topography and high pore fluid pressure lower the overpressure necessary for inducing failure. These findings could help in gaining insights on the internal state of the volcano and, hence, in advancing the assessment of the likelihood of volcano unrest.

## 1. Introduction

Ground deformation and other surface observations in volcanic areas are often interpreted in terms of overpressure changes in magma chambers. Since they represent the storage of magma rising from depth, and their growth and evolution regulate the transport of magma toward the surface, investigations on magma chamber stability are of particular interest. When a magma chamber is present, the condition for the onset of an eruption is the failure of its walls. To assess the likelihood of a volcanic eruption, it should be determined if the chamber is prone to rupture (Gudmundsson, 2006). The definition of the conditions for chamber failure is therefore of primary interest in understanding the factors that lead to chamber wall failure, possibly triggering an eruption. The magma overpressure that the wall of a chamber can sustain before failing depends complexly on the medium rheology, the mechanical rock properties, the stress distribution around the chamber, the volcanic edifice loading, and the source geometry (Sartoris et al., 1990; Grosfils, 2007; Hurwitz et al., 2009; Long and Grosfils, 2009; Martí and Geyer, 2009).

When failure conditions are investigated, a preliminary analysis of stability is required and total pressures, rather than simple internal overpressures, have to be considered (Grosfils, 2007; Gerbault et al., 2012). Usually simple elastic models, which represent the magma chamber as a cavity within an unloaded elastic medium, have been used to estimate wall ruptures (Gudmundsson, 2006; Pinel and Jaupart, 2004; Martí and Geyer, 2009), without making any assumptions about the total pressure acting on its wall. Recently, Grosfils (2007) demonstrated that the estimate of the value of the overpressure for tensile failure using gravitationally loaded models can be considerably different than that estimated using unloaded elastic models. The location and occurrence of ruptures was estimated by determining the regions where the elastic stresses exceed the tensile strength of the rocks. In volcanic regions many factors make the rocks deviate from elastic behavior and may strongly affect the estimate of source overpressure. A more realistic representation of rocks surrounding a magma chamber would be as brittle material in which fracture is controlled by a brittle failure criterion that is dependent on the net

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three-dimensional stress field adjacent to the chamber. Wang and Dusseault (1991a,b) explored fracture initiation in brittle material, where the rock is linearly elastic until the yield criterion is reached, weakens instantly and thereafter behaves perfectly plastically. Hydrofracture modeling (Wang and Dusseault, 1991a,b; Amadei and Stephansson, 1997; Gerbault et al., 2012), invoked to simulate chamber failure (Gudmundsson, 2006), and laboratory experiments on rock samples shows that shear failure (mode-II) precedes the occurrence of tensile failure (mode-I). The fracture process is influenced significantly by confining pressure (He et al., 2012). Rock displays splitting fractures under uniaxial compression, shear fractures under moderate confining pressure and fault zones or plastic flow under high confining pressure (He et al., 2012). The compression is mainly provided by gravity loading of the rock itself in natural conditions. Shear failure of the chamber wall may even affect tensile failure by reducing the tensile strength of the rock (Amadei and Stephansson, 1997). Thus, to model the processes associated with rock fracturing around a magma chamber, shearing mechanisms should be also taken into account (Gerbault et al., 2012) and could give insight on volcano-tectonic events accompanying magmatic unrest (Lengline et al., 2008; Martì et al., 2013). Moreover, a transition from brittle failure to thermally activated ductile flow may occur due to high temperature fields that may develop when periodic episodes of magma supply occur (Civetta et al., 2004; Annen et al., 2008). Therefore, the use of elastic behavior may not be appropriate to simulate the deformation and stress fields of rocks surrounding a magma chamber. This restriction can become important when trying to reproduce the observed deformation at the ground surface for inferring the internal overpressure of the magma chamber. In the absence of stress monitoring, an inappropriate modeling of the expected deformation could lead to erroneous estimates of the overpressure and the chamber size (Gudmundsson, 2006; Del Negro et al. 2009; Currenti et al., 2010; Newman et al., 2006).

Models invoking non-elastic rheologies (e.g., Bonafede et al., 1986; Dragoni and Magnanensi, 1989; Newman et al., 2006; Jellinek and DePaolo, 2003; Trasatti et al., 2005; Del Negro et al., 2009; Currenti et al., 2010) have been recently applied to reproduce ground surface deformation, but few efforts have been directed to test the circumstances that will promote magma chamber failure using these rheologies.

To investigate the complex interaction between magma chamber expansion and the inelastic behavior of host rock, we performed a stress–strain analysis using the Finite Element Method (FEM) for understanding, in general terms, the main factors controlling chamber failure. We analyze the computed stresses in both the brittle and viscoelastic regimes to predict the occurrence and location of mechanical failure of the chamber wall for different geometries, topographies, fluid pressure, rheologies, and reference stress state conditions, all factors that cannot be taken into account using standard analytical models. Gravitational loading and non-elastic rheologies are fully integrated in the FEM modeling. The use of elastoplastic and viscoelastic constitutive laws in a fully gravitational loading model provides the opportunity to explore the mechanical response of a volcanic rock mass to stress changes induced by magma overpressure. The numerical approach could advance the assessment of the likelihood of chamber failure and, hence, the understanding of unrest periods.

## 2. Numerical model

### 2.1. Rock rheology

When a rock is strained beyond the elastic limit, Hooke's law no longer applies. The behavior of rocks beyond their elastic limit is rather complicated. The lithosphere may have very different mechanical behavior, depending on rock composition, temperature, and pressure conditions. In the upper part of the crust, temperature and pressure are relatively low and the mechanical behavior is mainly driven by brittle failure, represented by a pressure-sensitive and temperature insensitive behavior. Active volcanoes host abundant earthquake activity. The seismicity usually recorded around a magma chamber confirms that the largest volume surrounding the chamber is undergoing brittle failure. In particular, volcano-tectonic earthquakes are shear failure events and can be used as indicators of the stress state of the volcano (McNutt, 1996; Lengline et al., 2008; Martì et al., 2013).

Several yield criteria have been investigated to model the mechanical behavior of rocks undergoing shear failure (Fung, 1965; Ranalli, 1995; Jaeger et al., 2007). Such failure depends on both the deviatoric stress and the overburden stress through the friction coefficient. The Drucker–Prager failure law can be properly used for modeling brittle behavior (Cattin et al., 2005; Cianetti et al., 2012; Apuani et al., 2013; Got et al., 2013) in the upper crust. The yield function for Drucker–Prager failure in the case of no hardening (perfect plasticity) may be written in terms of the second invariant of the deviatoric stress tensor ($J_2$) and the first stress invariant ($I_1$):

$$F = K + \alpha I_1 - \sqrt{J_2} \tag{1}$$

Generally, when the stress satisfies the yield criterion $F = 0$, the material will undergo plastic deformation. The Drucker–Prager criterion represents a smoothed version of the Mohr–Coulomb frictional failure criterion for a three-dimensional case. The coefficients $K$ and $\alpha$ may be related to the Mohr–Coulomb frictional failure properties cohesion ($c$) and friction angle ($\phi$), derived from laboratory experiments (Jaeger et al., 2007; Chen, 1982):

$$K = \frac{6c\cos\phi}{\sqrt{3}(3-\sin\phi)} \qquad \alpha = \frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)} \tag{2}$$

When $\alpha$ approaches zero the failure criterion reduces to the von Mises criterion, which could be applied to simulate rock behavior at high pressure and high temperature. The von Mises criterion was originally devised to study plasticity in metal and does not account for the experimental observation that yield stress of most rocks increases with increasing mean normal stress (Jaeger et al., 2007; Mazzini et al., 2009). To simulate the increase in rock strength with confining pressure the contribution of $I_1$ in the failure criteria is therefore considered (Haismon, 2006; Liu et al., 2004; Jaeger et al., 2007). Moreover, where temperatures are higher, the rock may transition to ductile behavior. With increasing temperature, the failure behavior changes from brittle fracture to ductile flow. The high temperatures around a heated magmatic source cause the rocks to undergo ductile flow instead of brittle failure in response to an internal pressure change. The hot region immediately surrounding the magma chamber is limited in size and most of the volcano is essentially cold. Therefore, the Drucker–Prager criterion may apply to most of the edifice where brittle failure takes place, but not where hot rocks surround the magma chamber. For rocks in the ductile regime, viscoelastic relaxation processes should be taken into account to model the mechanical response of the crust around the chamber (Del Negro et al., 2009; Newman et al., 2006; Dragoni and Magnanensi, 1989). This thermally activated ductile process results in viscoelastic stress relation within a shell surrounding the chamber and is simulated in our models by a viscoelastic Generalized Maxwell rheology of one spring-dashpot branch in parallel with a spring (Ranalli, 1995; Del Negro et al., 2009).