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Resonant tidal excitation of internal waves in the Earth's fluid core

Robert H. Tyler [a,b,*], Weijia Kuang [b]

[a] Department of Astronomy, University of Maryland, College Park, United States
[b] Planetary Geodynamics Laboratory, NASA Goddard Space Flight Center, Code 698, Greenbelt, MD 20771, United States

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ABSTRACT

It has long been speculated that there is a stably stratified layer below the core-mantle boundary, and two recent studies have improved the constraints on the parameters describing this stratification. Here we consider the dynamical implications of this layer using a simplified model. We first show that the stratification in this surface layer has sensitive control over the rate at which tidal energy is transferred to the core. We then show that when the stratification parameters from the recent studies are used in this model, a resonant configuration arrives whereby tidal forces perform elevated rates of work in exciting core flow. Specifically, the internal wave speed derived from the two independent studies (150 and 155 m/s) are in remarkable agreement with the speed (152 m/s) required for excitation of the primary normal mode of oscillation as calculated from full solutions of the Laplace Tidal Equations applied to a reduced-gravity idealized model representing the stratified layer. In evaluating this agreement it is noteworthy that the idealized model assumed may be regarded as the most reduced representation of the stratified dynamics of the layer, in that there are no non-essential dynamical terms in the governing equations assumed. While it is certainly possible that a more realistic treatment may require additional dynamical terms or coupling, it is also clear that this reduced representation includes no freedom for coercing the correlation described. This suggests that one must accept either (1) that tidal forces resonantly excite core flow and this is predicted by a simple model or (2) that either the independent estimates or the dynamical model does not accurately portray the core surface layer and there has simply been an unlikely coincidence between three estimates of a stratification parameter which would otherwise have a broad plausible range.

1. Introduction

The region of the core-mantle boundary (CMB) is central in controlling the flux of energy and momentum into or out of the Earth's fluid core and has therefore been of great interest in a wide variety of geophysical studies. Various conjectures and observational inferences have been made regarding the structure of this region and in particular there has been debate over the mass density stratification of a thin layer near the top of the fluid core and the associated dynamical implications and constraints. It has been understood since long that this stratified layer will support internal waves, but the conditions for the resonant forcing of these waves by tidal forces has not been adequately addressed. The purpose of this paper is to provide these conditions using dynamically consistent solutions of the full Laplace Tidal Equations (LTE). A preliminary matter is to review proposed estimates or constraints describing the stratification parameters of this layer, as these parameters control the propagation speed (s) of the internal waves, and thereby the degree of resonance.

The stratified layer was initially proposed from geochemical experiments (Higgins and Kennedy, 1971) and consideration of the thermo-chemical evolution (Gubbins et al., 1982). While these and closely related later studies have been important in the postulation of the stratified layer, usefully constrained estimates of the stratification parameters were not obtained. The results can be collectively summarized as the expectation that the Brunt–Vaisala (buoyancy) frequency N is typically of the order of the rotation frequency (or less), and that the thickness h of the stratified layer is several 10's to 100's of kilometers. Within continuous-stratification models of the assumed dynamics, one might estimate the internal wave speed as $c \approx Nh$ (e.g. Braginsky, 1998), implying that c is several 10's m/s or less. Within a two-layer model under the limit where the lower layer is much larger than the upper layer (a so-called 1.5-layer model such as adopted in this study), $c = (g'h)^{1/2}$. This is the same expression as the shallow-water wave speed of a

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* Corresponding author at: Planetary Geodynamics Laboratory, NASA Goddard Space Flight Center, Code 698, Greenbelt, MD 20771, United States. Tel.: +1 (301) 614 6472; fax: +1 (301) 614 65224.

E-mail address: robert.h.tyler@nasa.gov (R.H. Tyler).

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thin, homogenous fluid, with the exception that the gravitational acceleration g has been replaced by the "reduced gravity" $g' = \frac{\Delta\rho}{\rho} g$, where $\Delta\rho/\rho$ is the fractional density difference between the two layers. Estimates of c using this approach, where possible, provide results roughly consistent with the above. A recent study (Gubbins and Davies, 2013) of a stratified layer maintained by barodiffusion of oxygen, sulfer and silicon provides an estimate for N reaching $N = 1.5 \times 10^{-3}\ \mathrm{s}^{-1}$ and $h = 100$ km, from which $c \approx Nh = 150$ m/s.

Further early attempts to describe the stratified layer came through studies of the geomagnetic secular variation and the convection required in driving the geodynamo. Inversion of observations of geomagnetic secular variation provides a description of flow at the top of the core from which constraints on the description of this stratified layer have been obtained (e.g. Whaler, 1980). Inferences drawn from this type of study are, however, highly contingent on the approximations and specific assumptions used in the core flow inversion, and on the limited geomagnetic data. Inferences have also been drawn through the operation of numerical dynamo simulations involving the stratified layer (Nakagawa, 2011; Stanley and Mohammadi, 2008). In such studies, the layer thickness and the density difference are arbitrary, and their dynamical consequences are clearly shown in the properties of the generated magnetic field at the CMB. But it is also understood that any inference of the realistic situation is indirect because the parameter domains in these numerical simulations are very limited and far away from that of the Earth's core. While these dynamo simulations provide qualitative information of the process of the geodynamo and perhaps its dependence on the stratified layer, they do not yet provide strong quantitative constraints describing the realistic properties of the layer.

Stratification parameters have been obtained more recently from analyses of seismic wave forms. Using the reduced-gravity approach together with the values described in (Lay and Young, 1990) of $\Delta\rho/\rho \sim 0.01$ and $h \sim 50 - 100$ (km), and assuming $g \approx 10.7\ \mathrm{m/s^2}$ (Braginsky, 2000), produces $c \sim$73–103 (m/s). A very recent study (Helffrich and Kaneshima, 2010) indicates the presence of a substantially thick stably stratified layer in the fluid core directly below the CMB. In supplementary online information provided with the cited study, the layer (h) was estimated to be 300 km thick and 7.5 parts per thousand less dense than the fluid below (i.e. $\Delta\rho/\rho \sim 0.0075$). These parameters produce $c = 155$ m/s.

This tidal energy transfer rate will be greatly enhanced if internal waves in the stratified layer are resonantly excited by tidal forces. For this to occur, there must be a near match in both frequency and spatial pattern between at least one of the tidal forces and at least one of the stratified layer's natural modes of oscillation. While the tidal forces may be prescribed from astronomical parameters, the layer's natural modes depend on the stratification parameters. To discover if there is a match we may then, for each of the important tidal force constituents, take the spatial structure and frequency as prescribed and calculate the stratification parameters that would be needed to produce a resonant response to the tidal force. More specifically, the approach followed in this study is to treat the stratification parameter c as a free parameter, and through repeated solution we construct a description of the dependence of the tidal response on this parameter. This theoretical description of the stratification dependence of the transfer rate of tidal energy to the stratified core is the primary independent result of this study. Secondly, by comparing these results with the stratification estimates reviewed above, we draw inferences about the tidal state in the CMB layer. These second results obviously depend on the validity of the stratification-parameter estimates which show the range described above. We note that the two most recent estimates show a remarkably similar result for c obtained using quite different methods ($c = 155$ m/s (Helffrich and Kaneshima, 2010), and $c = 150$ m/s (Gubbins and Davies, 2013)). While we do not think this agreement provides confirmation of the parameter value, we shall take from this $c = 1.5 \times 10^2$ m/s as representing the best current estimate.

The outline of this paper is as follows: The following Section 2 is somewhat lengthy and primarily directed and providing the formulation and intuition behind the use of the 1.5-layer model. While this formulation is widely used in the dynamics of the stratified ocean and atmospheres, it is perhaps less well understood among the community investigating core flow. Those readers familiar with the class of "equivalent barotropic" models used to describe the dynamics of stratified fluids, might scan or skip this section. The section aims to describe the assumptions by which the Laplace Tidal Equations for an unstratified fluid (Hough, 1898) can be extended to the stratified case by simply changing the interpretation of some variables and parameters: For the 1.5-layer version chosen to model the dynamics within the stratified layer below the CMB, the shallow-water wave speed c is replaced by the reduced-gravity version (as described above in this section), the fluid momentum vector refers to only the flow in the thin upper layer, and the consequence of tangential flow convergence is to depress the lower interface of the layer (rather than lift the upper surface). The method for solving the LTE can be summarized here as a semi-analytical method that analytically decomposes the LTE into spherical-harmonic bases and performs a numerical inversion to obtain the associated coefficients. The results obtained using this model and method are described in Section 3. These theoretical results for the tidal solution are presented as a function of two unknown parameters. The imposition of potentially realistic parameters to select the appropriate solution scenario is performed in Section 4, and conclusions from this work are described in Section 5.

2. Model and method

The calculation of the required conditions for resonance is accessed through a mathematical model of the assumed dynamics in the surficial core. The *model* comprises these dynamical equations and assumptions. The *method* prescribes the strategy for obtaining solutions from the model. The equations assumed to govern the tidal dynamics within the thin layer are the Laplace Tidal Equations (LTE) including tidal forcing and dissipation terms. The LTE can be regarded as statements of conservation of momentum and mass appropriate for a thin layer. The name is of historical interest as these equations are not restricted to tidal applications. Indeed, the LTE are largely equivalent to the horizontal components of the linearized Navier–Stokes equations which show the inertial + rotational accelerations balanced by a pressure gradient, plus force and dissipation terms. What distinguishes the LTE from the linearized Navier–Stokes equations is the assumption that the pressure function involved can be described such that the vertical and horizontal components of the momentum equation are separable. This separability is required in Classical Tidal Theory (Lindzen and Chapman, 1969), and more generally forms the basis in geophysical fluid dynamics for separating external and internal modes. The separation constant between the LTE and the equation (s) describing the vertical structure are the wave speeds $c_{(i)}$ associated with the suite of modes indexed as (i). (In the approach here there is only one mode (the first baroclinic mode) retained, and therefore the index (i) is discarded.) Because c shall be treated as a free parameter, the LTE solution set calculated from the plausible range of c then provides a complete set of tidal scenarios that is independent of the specific assumptions regarding the vertical structure. Indeed a variety of layer (as in the case used in this study) or continuous-stratification models may be applied for the