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# On the scaling of heat transfer for mixed heating convection in a spherical shell

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ABSTRACT

Planetary mantles and solid shells of icy satellites potentially undergoing natural convection are subjected to a mixed heating configuration including basal (from thermal exchanges with a subjacent, possibly liquid, layer) and internal (from radioactive decay or tidal dissipation) sources. In the quasi-static approximation, the average cooling/heating of the layer is also considered as an instantaneous internal heat source to model transient evolutions. In a previous study (Choblet and Parmentier, 2009), we have proposed simple scaling relationships to describe heat transfer for an isoviscous fluid in such a mixed heating configuration in the case of a Cartesian geometry. Here, we extend this analysis to the case of a spherical shell. A framework based on a temperature scale associated with the global surface heat flux is introduced. This enables a simple description of the cold boundary layer, independent of the heating configuration and of the relative radius of the inner boundary of the shell. When free-slip mechanical boundaries are prescribed, numerical experiments present a significant departure from the prediction (up to ≃30%). We show that this is caused by the impact of hot plumes on the cold boundary layer when a large amount of basal heating is prescribed. The results of no-slip calculations are well predicted by the scaling which thus could be applied to planetary mantles where convection occurs beneath a rigid lithosphere. The lower hot boundary layer is included in our analysis through the ratio of the temperature differences across both boundary layers: the simple scaling indicates that this ratio is independent of the Rayleigh number, and varies only with the amount of basal heating and with the curvature of the layer. This is shown to be valid in the no-slip case. In the free-slip case, a departure from this scaling is observed in the calculations but for the range of values corresponding to planetary bodies, the agreement is good. We conduct transient numerical experiments and show that the quasi-static approximation is valid in the configuration investigated here. Implications for more complex planetary set-ups are discussed.



## 1. Introduction

Subsolidus thermal convection in planetary mantles is characterized by a mixed heating configuration:

(i) Volumetric heating is caused by radioactive decay and/or tidal dissipation (other sources such as the gravitational energy associated with the formation of the crust of terrestrial planets are probably minor contributions, see, for the Earth, Jaupart et al., 2007). Both sources can be strongly heterogeneously distributed: in the case of the Earth's mantle, concentration gradients for radiogenic elements may arise from melting processes (e.g., Coltice and Ricard, 1999) or due to the long term preservation of chemically dense reservoirs (e.g., Tackley, 2002; Le Bars and Davaille, 2004); patterns of tidal dissipation strongly depend on the orbital configuration of the planet/satellite (e.g., Běhounková et al., 2010). In addition, in the quasi-static approximation, secular cooling can be considered as a volumetric heat source (e.g., Krishnamurti, 1968a,b; Daly, 1980; Choblet and Sotin, 2000): for some configurations, transient evolutions may be described as series of thermal equilibriums if the average cooling (heating) of the layer is considered as a uniform volumetric heat source (sink).

(ii) Geochemistry of siderophile elements indicates that the formation of the metallic core within terrestrial planets is a rapid process (e.g., Li and Agee, 1996, 2006) which has been described in a recent model as a possibly catastrophic event (Ricard et al., 2009). This in turn suggests that a significant fraction of the energy associated with this differentiation could contribute to the heating of the newly-formed iron-rich core. The cooling and solidification of the core is then a primary source of heating from below for thermal convection in the mantle. Heat flux out of the core also controls its

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solidification rate along with the strength of the magnetic dynamo (e.g., Labrosse et al., 2001,). For differentiated icy satellites, basal heating of convecting icy layers could be associated with radiogenic heat in a silicate core and with the thermal evolution of a deep internal ocean, the resulting temperature distributions in the convecting icy layer determining whether and how much of this layer will be liquid (e.g., Tobie et al., 2003, 2008).

For both terrestrial planets and icy satellites, the relative proportions of heating from below and volumetric heating result in variations of mantle temperature distributions that should be reflected in the style of volcanism, from one dominated by local hot spots at the top of rising plumes to more globally distributed extrusions perhaps controlled by lithospheric structure. In a previous study (Choblet and Parmentier, 2009), we have shown that parameter $q^*$ measuring the fraction of the basal heating in the global heat power lost through a planet's surface ultimately controls the radial temperature profile in the mantle. $q^*$ naturally evolves during the planet's history and is also expected to differ among the planetary bodies considered.

Our preceding study (Choblet and Parmentier, 2009) considered a Cartesian geometry (so that in this simple case $q^* = q_b/q_t$, with $q_\bullet$ referring either to the surface ($_t$) or the basal ($_b$) heat flux). This model could approximate the configuration of outer crusts of icy satellites whose thickness ($R_t - R_b$) is small compared to the radius of the body ($R_t$). Among terrestrial planets, the rocky mantle of Mercury whose core is relatively large ($f = R_b/R_t \sim 0.8$, cf. e.g., Spohn et al., 2001) could also be reasonably well described by this simplified geometry. Other terrestrial planetary mantles however present a significantly curved geometry ($f \sim 0.4 - 0.6$). The freezing/melting history of the outer part of icy satellites can also lead to significant temporal variations of $f$ for some of these bodies (in the case of Saturn's largest moon, Titan, see e.g., Tobie et al., 2006). If $f$ is introduced in the definition of the basal heat fraction, $q^* = f^2 q_b/q_t$, it is evident that for a given amount of basal heat power, the geometry naturally leads to a higher basal heat flux as the inner radius is smaller. This can be expected to induce larger buoyancy in the hot thermal boundary layer and favor hot instabilities.

The effect of curvature on basally heated ($q^* = 1$) convective heat transfer has been studied from two-dimensional numerical experiments by Vangelov and Jarvis (1994) and Jarvis (1995), and later on from three-dimensional models by Wolstencroft et al. (2009). The effect of the addition of a varying amount of internal heating ($0 \leqslant q^* \leqslant 1$) was proposed by Sotin and Labrosse (1999) on the basis of 3D Cartesian computations and tested and refined by Shahnas et al. (2008). Deschamps et al. (2010) recently provided a precise synthesis from additional 3D spherical calculations and proposed scaling relationships relating internal temperature as well as surface and basal heat fluxes to the convection parameters. Such relationships were inverted from all the available numerical experiments and accurately fit the various results of differing studies. These relationships can therefore be used to investigate parameterized evolutions as done for example in the pioneering work by Stevenson et al. (1983). At present, it shall be noted that, despite their known limitations (see, in the case of the cooling of the Earth, Jaupart et al., 2007), parameterized models nevertheless still constitute a convenient tool for the first insights into the thermal history of planetary bodies: despite the constant extension of computational capabilities, 3D investigations of such evolutions are necessarily restricted to a limited number of parameters. An interesting compromise is the 2D approach that enables a treatment of the planetary history which is both more realistic than what 1D parameterizations allow, and reasonably close to 3D models (see, for example, for the outgassing of terrestrial planets, Xie and Tackley, 2004 for the Earth's thermal history, Nakagawa and Tackley, 2010).

The present work extends the Cartesian analysis presented in Choblet and Parmentier, 2009 to the spherical geometry. Its goal is not to provide more precise coefficients for the power–law scalings but to propose a mechanical interpretation of such coefficients. The Rayleigh number and the partitioning of heat sources into basal and volumetric heating are considered as free parameters and simple scaling relationships are provided in order to guide the interpretation of both more complex models and the uncertain observations of planetary bodies.

Here our Cartesian analysis (Choblet and Parmentier, 2009) is extended to the case of a spherical shell of geometrical factor $f$. Again, the framework we propose focuses on a temperature scale based on the global surface power $F_t = 4\pi R_t^2 Q_t$ and thus differs from the above mentioned studies. A simple model equivalent to the thermal boundary layer analysis developed by Jarvis (1995) (or with a slightly different approach by Sotin and Labrosse (1999)) is introduced in this framework. Both steady-state and transient additional 3D calculations are presented and departures of these results from the simple prediction, already noted by Shahnas et al. (2008) and Deschamps et al. (2010), are interpreted in terms of the interactions of thermal plumes emanating from one of the boundary layers with the opposite boundary layer. The nature of the mechanical boundary condition is shown to strongly control these departures from the simple model.

## 2. Mixed heating convection in a spherical shell

### 2.1. Global heat balance

Convection of an isoviscous fluid in a spherical shell is considered with both volumetric and basal heating. The radii of the bounding spheres are $R_t$ (top) and $R_b$ (bottom) and their ratio is noted $f = R_b/R_t$. The global balance for heat is therefore

$$Q_t R_t^2 = Q_b R_b^2 + H(R_t^3 - R_b^3)/3 \quad (1)$$

where $Q_\bullet$ refer to the heat flux through the spherical boundaries ($_\bullet$ denotes either the top $_t$ or bottom $_b$ boundary) and $H$ is the volumetric heating rate.

### 2.2. Dimensionless formulation

Characteristic scales are used to introduce a dimensionless framework: the shell thickness $d = R_t - R_b$ for length and the diffusive scale $d^2/\kappa$ for time where $\kappa$ is thermal diffusivity. For example, $\delta_\bullet = d_\bullet/d$ (where $_\bullet$ is either $_t$ or $_b$) refer to the dimensionless values of the thicknesses of the top and bottom boundary layers of dimensional thickness $d_\bullet$. Two scales can be used to characterize the temperature variations within the fluid layer. While, for a similar problem, Sotin and Labrosse (1999) and Shahnas et al. (2008), for example, introduce the classical scale $\Delta T^T = T_b - T_t$ based on the global temperature difference, Choblet and Parmentier (2009) propose the use of $\Delta T^Q = Q_t d/k$, based on the surface heat flux $Q_t$ (with $k$ the thermal conductivity). In the following, both temperature scales are used. For example, when temperature scale $\Delta T^T$ is used, $\theta_\bullet^T = \Delta T_\bullet/(T_b - T_t)$ are the dimensionless values of the temperature differences across the boundary layers (with dimensional temperature variation $\Delta T_\bullet$) while, when $\Delta T^Q$ is used, $\theta_\bullet^Q = k\Delta T_\bullet/dQ_t$. Similarly, $q_\bullet^T = Q_\bullet d/k\Delta T_t$ and $q_\bullet^Q = Q_\bullet/Q_t$ are the dimensionless heat fluxes. Note that as a consequence $q_t^Q = 1$. Table 1 lists the various notations associated with these two scales (see the derivations in Appendix A).