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The influence of thermo-compositional boundary conditions on convection and dynamos in a rotating spherical shell

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ABSTRACT

Today's geodynamo is driven by a combination of secular cooling and of latent heat and light core constituents emanating from a growing inner core. The early dynamos of Earth and Mars, however, functioned without an inner core and were thus exclusively driven by secular cooling. Dynamo simulations model secular cooling by internal buoyancy sources and the inner core-related driving by bottom sources. Adopting a codensity approach, we explore how the different combination of thermo-compositional boundary conditions and source distributions affects nonmagnetic convection and dynamo simulations. The impact of the outer boundary condition, fixed codensity or fixed codensity flux (temperature or heat flux when no compositional contribution is present), is only large when the convection is mainly driven by internal sources. When bottom sources dominate, the lower boundary condition becomes more important. In both cases, a fixed flux condition promotes larger convective scales than a fixed codensity condition. A magnetic field can further increase the flow scale and is important to obtain large-scale structures at high Rayleigh numbers. The thermo-compositional outer boundary condition thus plays an important role for the early dynamos in Earth and Mars. Using the more realistic fixed flux condition promotes dipole dominated fields here. For today's geodynamo, however, the lower boundary condition may be more influential.

1. Introduction

Though today's numerical dynamo simulations successfully model many features of the planetary magnetic fields, they also rely on several unrealistic simplifications (e.g. Christensen and Wicht, 2007; Stanley and Glatzmaier, 2010). As the models are refined, the convective driving mechanism and the related boundary conditions are reconsidered. There are two potential sources to drive convection in the dynamo regions of terrestrial planets that we will focus on here. The light elements (sulfur, oxygen, silicon) which are mixed into the liquid iron/nickel core are less compatible with the solid state. They are thus emanating at the front of a growing inner core and lead to compositional convection. The latent heat released upon inner core freezing, secular cooling and, probably only in a subordinate degree, radiogenic heating drive thermal convection (e.g. Nimmo, 2007).

Numerical simulations solve for convection and magnetic field generation in a rotating spherical shell that represents the liquid outer core. Secular cooling and radiogenic heating are typically represented by internal heat sources. Temperature boundary conditions and, when a growing inner core is present, also compositional boundary conditions have to be implemented in a way that is consistent with the convective driving. At the top boundary, the influence of the rocky mantle needs to be considered. Because of the vastly different viscosities of the liquid iron core and the rocky mantle, changes in the two systems occur on very different time scales which are in the order of decades to centuries in the vigorously convecting core but amount to tens of million years in the mantle. The structure of the thermal boundary layer on the mantle side controls the heat flux through the core mantle boundary (CMB), which can be treated as constant on the time scales of core dynamics. A fixed heat flux rather than a fixed temperature boundary condition is thus more appropriate at the outer boundary of numerical dynamo models for terrestrial planets. Since the light core material cannot penetrate the mantle at any significant rate, a vanishing flux is the most realistic outer boundary condition for the compositional component of the buoyancy.

Ideally the heat flux variation at the core–mantle boundary is linearly related to the seismic S-velocity perturbations of the lowermost mantle, as determined by seismic tomography. The relation is more complex when chemical heterogeneity and the effects of the post-perovskite phase transition are accounted for, but a generally positive correlation exists also in this case (Nakagawa and Tackley, 2008). Mantle convection simulations can help to constrain the lateral variation of CMB heat-flux for other planets.

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At the inner boundary neither fixed temperature nor fixed flux boundary conditions seem realistic. The local solidification rate determines the local compositional and the local latent heat flux and both are proportional to each other. The solidification rate in turn depends on the rate of change in mostly the temperature, but also in composition, and in pressure. This leads to conditions that tie the heat flux and the compositional flux to the local convection dynamics (Braginsky and Roberts, 1995) which have so far only been implemented in the dynamo model by Glatzmaier and Roberts (1996). However, these authors did not explore the possible implication of this more realistic approach compared to the classical implementations of either fixed temperature or fixed flux conditions.

Most dynamo simulations use simplified models which are motivated to a great deal by their numerical convenience. Thermal and compositional density variations are frequently combined into one variable called codensity (Braginsky and Roberts, 1995). Only one evolution equation is then solved to describe its dynamics which effectively means that temperature and composition are assumed to obey the same transport properties. Though their molecular diffusivities differ by several orders of magnitude, it is argued that the turbulent diffusivities used to parameterize the unresolved small scale turbulence may effectively be similar. Such an approach leaves only one condition at each the inner and the outer boundary. Concerning the convective driving the ratio of bottom to internal sources is the only additional characterizing parameter. For simplicity, the authors of such models often only talk about temperature effects though the driving sources are largely indiscernible. We also adopt a codensity model and follow this notion in our analyses.

Using a codensity approach, several authors investigated planetary dynamos by MHD simulations, in which fixed codensity conditions (corresponding to fixed temperature when no compositional contribution is present) have been commonly assumed (e.g. Kageyama and Sato, 1995; Christensen et al., 1999; Sreenivasan and Jones, 2006; Takahashi et al., 2008). Others employed a fixed codensity flux condition (corresponding to fixed heat flux without compositional contributions), but mostly focused on the impact of different lateral variation patterns. The results suggest that the pattern can for example influence the reversal behavior (Glatzmaier et al., 1999; Olson et al., 2010), cause hemispherical differences in the secular-variation signature (Christensen and Olson, 2003), and may also explain the seismic anisotropy observed in Earth's inner core (Aubert et al., 2008). Stanley et al. (2008) demonstrated that it could also explain the magnetic dichotomy observed on Mars where the crust is much more strongly magnetized in the southern than in the northern hemisphere. However, the question whether the boundary condition, fixed codensity or fixed codensity flux, makes a fundamental difference has been addressed in a few studies only.

Even a homogeneous codensity flux condition at the outer boundary can lead to interesting differences compared to the more conventional fixed codensity condition. The flux condition possibly has two immediate consequences for the convective flow pattern: it allows stronger zonal flows to develop and promotes larger non-axisymmetric flow structures. The stronger zonal flows are thermo-compositional winds (thermal winds when no compositional contribution is present) which rely on latitudinal codensity (temperature) variations. These variations are forced to zero at the outer boundary when fixed codensity conditions are used, which explains the weaker thermo-compositional winds in this case. However, even when flux conditions are employed the kinetic energy carried by zonal flows amounts to only a few percent of the total kinetic energy (Christensen and Wicht, 2007; Hori et al., 2010). Thus the impact on the non-axisymmetric flow components seems more important.

That convective features become larger when employing the flux condition has already been reported for classical non-rotating Rayleigh-Bénard convection, where only thermal buoyancy is included (e.g. Jakeman, 1968; Chapman and Proctor, 1980; Ishiwatari et al., 1994). When temperature boundary conditions are used, the convective cells tend to assume a horizontal dimension which is similar to the layer thickness. Flux boundary conditions, however, promote much wider features because the temperature is allowed to vary at the boundaries. Chapman and Proctor (1980) investigate nonlinear convection in a box with flux boundary conditions and find that any broad container will ultimately contain only one convective roll when the Rayleigh number is larger than the critical value for the onset of convection.

In a dynamo, Coriolis forces and Lorentz forces also influence the flow scale. In the case of rapidly rotating convection, the Coriolis force dominates the force balance and imposes a quasi two-dimensional so-called geostrophic structure. The impact of the Coriolis force is typically quantified by the Ekman number $E = \nu/(\Omega L^2)$ with ν being the kinematic viscosity, Ω the rotation rate and L the characteristic length scale, which is an estimate for the ratio of viscous to Coriolis forces in the Navier–Stokes equation. In order to facilitate the onset of convection a small part of the Coriolis force must be balanced by viscous forces. The azimuthal flow scale therefore decreases with the Ekman number to keep viscous effects efficient enough. Linear stability analyses (e.g. Chandrasekhar, 1961) show that the flow length scale decreases like $E^{1/3}$ for sufficiently small E. At moderately small Ekman numbers $E > 10^{-4}$ a heat flux condition still leads to significantly larger flow scales at the onset of convection than a fixed-temperature condition (Takehiro et al., 2002; Busse and Simitev, 2006; Gibbons et al., 2007), but at lower Ekman numbers $E \lesssim 10^{-4}$ the respective Coriolis force effect dominates the influence of the flux boundary condition on the flow scale at the onset of convection. However, since the influence above the onset is not fully known, we come back to this problem for supercritical convection in Section 4.1. Furthermore this influence may change in the presence of magnetic field (Sakuraba (2002); see below 2).**

If a magnetic field is present, the Lorentz force can help to balance the Coriolis force. The ratio of Lorentz to Coriolis force in the Navier–Stokes equation is usually quantified by the Elsasser number $\Lambda = B^2/(\rho\mu_0\lambda\Omega)$, with B being the characteristic magnetic field strength, ρ the density, μ_0 the magnetic permeability and λ the magnetic diffusivity. If Λ is of order one, viscous forces are no longer needed to balance the Coriolis force and the flow can become large scale, up to the dimension of the layer thickness or shell thickness. A magnetic field can thus help the convection to get started and may decrease the critical Rayleigh number. The respective effects are predicted by the linear theory of magnetoconvection where the magnetic field is imposed and observed in magnetoconvection simulations (see reviews by Proctor (1994), Zhang and Schubert (2000), Jones (2007) and references therein). Self-consistent dynamo simulations in Cartesian geometry show the enlargement of the flow scale when viscous effects become small enough at $E < 10^{-4}$ (Rotvig and Jones, 2002; Stellmach and Hansen, 2004). Although some spherical dynamo simulations reached Ekman numbers down to $E = \mathcal{O}(10^{-6})$, the effects on the flow scale are less drastic in this case. Takahashi et al. (2008) report an increase of the mean length scale by 20% when comparing non-magnetic and dynamo simulations at $E = 2 \times 10^{-6}$.

Recent dynamo simulations in spherical geometries confirm the impact of buoyancy (thermal) boundary conditions on the flow scale. Including only thermal buoyancy, Sakuraba and Roberts (2009) demonstrate that convective flow structures become larger when they change the outer thermal boundary conditions from isothermal to constant heat flux in their dynamo simulations at $E \approx 2 \times 10^{-6}$. They attribute this effect to the stronger magnetic