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Numerical study on double-diffusive convection in the Earth's core

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ABSTRACT

Our numerical study focuses on convection in a rotating spherical shell with the objective to model combined thermal and compositional convection as proposed for the Earth's core. Since the core is cooling, a thermal gradient is established, which can drive thermal convection. Simultaneously, due to the solidification of the inner core latent heat is released at the freezing front and the concentration of the light constituents of the liquid phase increases thus providing a source for compositional buoyancy. Typically, the molecular diffusivities of both driving components differ by some orders of magnitude. To account for this difference it is indicated to adopt a double-diffusive convection model in treating Earth's core dynamics. As opposed to purely thermal or purely compositional convection the double-diffusive system is controlled by two Rayleigh numbers associated with the respective buoyancy sources. Using the Rayleigh numbers as control parameters neutral curves of the linear onset of convection in the rotating shell are determined for different Ekman numbers and diffusivity ratios. It is found that the neutral curves depend significantly on the system parameters. By comparison with the analytical solutions of the rotating cylindrical annulus it is shown that the neutral curves represent a superposition of curves associated with solutions for different azimuthal wave numbers. Furthermore, fully non-linear simulations are presented in order to elucidate the effect of isochemical and fixed chemical flux boundary conditions on the convection. We consider three driving scenarios with varying thermo-chemical forcing ratios. Both the forcing ratio and the chemical boundary condition have distinct effects on the system that are discussed separately.

1. Introduction

The Earth's outer core is assumed to consist of an electrically-conducting fluid, mostly molten iron and nickel. Additionally, it contains some fraction of light elements whose exact nature is not yet well constrained. Likely candidates are sulfur, silicon, oxygen, carbon, and hydrogen (Poirier, 1994). Heat is permanently extracted from the core causing it to gradually cool down and thereby establishing a thermal gradient which can drive thermal convection in the outer core. The cooling also controls the solidification of core material into the inner core which is thus constantly growing and releasing latent heat. As the matrix of the solid alloy constituting the inner core cannot incorporate the same fraction of light elements as in the liquid state, the excess composition is expelled and accumulates in the outer core liquid at the freezing front (Fearn, 1998). Thus, the fluid at the inner core boundary (icb) gains chemical buoyancy which is the source of chemical convection in the outer core. Compositional diffusivity is estimated to be some orders of magnitude lower than thermal diffusivity (Braginsky and Roberts, 1995) which leads to distinct differences in the dynamics of thermal and compositional convection.

In this numerical study we aim to investigate the nature of simultaneously thermally and chemically driven convection in the outer core. For this purpose we consider a double-diffusive convection model of a self gravitating, rotating spherical fluid shell. Double-diffusivity implies that separate transport equations for temperature and composition have to be introduced in order to account for the different diffusivities of the two components. The dependence of convection and the geodynamo in a spherical shell on the Prandtl number, i.e. the ratio of viscosity to diffusivity, has been investigated in previous studies (Tilgner and Busse, 1997; Simitev and Busse, 2003, 2005; Busse and Simitev, 2006; Sreenivasan and Jones, 2006; Šimkanin and Hejda, 2011). Thermo-chemical convection has recently been studied by Breuer et al. (2010) and its relevance for a dynamo model for Mercury's core has been proposed by Manglik et al. (2010). Laboratory experiments have been conducted by Cardin and Olson (1992). In the first part of this study we investigate the criticality of thermo-chemical convection within the Ra_T-Ra_C-space by solving the linearized set of equations, where Ra_T and Ra_C are the thermal and compositional Rayleigh number, respectively. This is done for Ekman numbers 10^{-3} and 10^{-4} and two different diffusivity ratios (see Section 4.1). Addition-

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ally, we compare our results with the linear solutions of the rotating cylindrical annulus recently studied by Simitev (2011). In the second part the fully non-linear thermo-chemical convection model is considered in order to examine the influence of different thermo-chemical driving scenarios on the properties of the convective flow (see Section 4.2). The Ekman number is set to 10^{-4} and a diffusivity ratio, often referred to as the Lewis number $L = \kappa_T/\kappa_C$, of 10 is adopted. We consider both fixed chemical flux and isochemical boundary conditions in order to test their respective effects on the flow dynamics.

2. Mathematical model

We consider a rotating spherical shell with an earth-like ratio of inner to outer radius of $a = R_i/R_o = 0.35$. The whole fluid shell is rotating with constant angular frequency Ω about the z-axis. The Boussinesq approximation is applied to the fluid; that is, all fluid material properties are assumed constant except for density changes in the buoyancy term. Density ρ is modeled as being a function of both the temperature T and the mass fraction of a light chemical constituent C. Thus, the governing equation of state can be formulated as

$$\rho = \rho_0[1 - \alpha(T - T_0) - \beta(C - C_0)], \tag{1}$$

where T_0, C_0, and ρ_0 are the reference temperature, composition, and density, respectively. α is the thermal expansivity and β an analogous compositional coefficient.

In the double diffusive case we have to solve for both transport equations separately, unlike the codensity approach, in which temperature and composition can be combined into one variable due to the choice of equal diffusivities (Braginsky and Roberts, 1995; Lister and Buffett, 1995; Shearer and Roberts, 1997; Christensen and Wicht, 2008; Aubert et al., 2008). In the dimensionless form the equations for the deviations of temperature ϑ and composition ζ from their respective reference states are given by

$$\partial_t \vartheta + \boldsymbol{u}\cdot\nabla\vartheta = \frac{1}{Pr_T}\nabla^2\vartheta \tag{2}$$

and

$$\partial_t \zeta + \boldsymbol{u}\cdot\nabla\zeta = \frac{1}{Pr_C}\nabla^2\zeta + S. \tag{3}$$

Here $\partial_t\cdot$ denotes the partial derivative $\frac{\partial\cdot}{\partial t}$. We use the shell gap $d = R_o - R_i$ as typical length scale and the viscous diffusion time d^2/ν as the typical time scale. ν denotes the kinematic viscosity of the fluid. The velocity field $\boldsymbol{u}$ is then scaled by ν/d. Temperature is scaled by its total difference ΔT across the shell. Composition is scaled as a function of the applied boundary conditions. When flux boundary conditions are involved ζ is scaled by $d^2\dot{C}_0/\nu$, otherwise the total difference ΔC across the shell is used. Furthermore, fixed flux conditions on the boundaries result in a time-dependent reference state, which manifests itself as a volumetric sink, $S = -1$, in the non-dimensional transport equation of ζ. The sink disappears, i.e. $S = 0$, when composition is fixed on the boundaries. For an analytical definition of $\dot{C}_0$ see Section 2.1. The transport equations also include two non-dimensional system parameters namely the thermal Prandtl number Pr_T and the compositional Prandtl number Pr_C. These are defined as

$$Pr_T = \frac{\nu}{\kappa_T} \quad \text{and} \quad Pr_C = \frac{\nu}{\kappa_C}, \tag{4}$$

the ratios of kinematic viscosity to thermal diffusivity κ_T and compositional diffusivity κ_C, respectively. The gravitational acceleration is assumed to increase linearly with radius r across the spherical shell. Within the rotating frame of reference the non-dimensional Navier–Stokes equation is given by

$$\partial_t\boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \frac{2}{Ek}\boldsymbol{e_z}\times\boldsymbol{u} + \nabla\Pi = \nabla^2\boldsymbol{u} + \left(Ra^T_{vis}\vartheta + Ra^C_{vis}\zeta\right)\frac{\boldsymbol{r}}{R_0}. \tag{5}$$

Here $\boldsymbol{e_z}$ is the unit vector in the z-direction and $\nabla\Pi$ is the modified pressure gradient containing all terms which can be written as gradients. Pressure itself is scaled by $\rho_0\nu\Omega$. The three additional dimensionless control parameters are Ekman number Ek, viscous thermal Rayleigh number Ra^T_{vis}, and viscous compositional Rayleigh number Ra^C_{vis}. The following definitions apply:

$$Ek = \frac{\nu}{\Omega d^2}, \quad Ra^T_{vis} = \frac{g_0\alpha\Delta T d^3}{\nu^2}, \quad Ra^C_{vis} = \frac{g_0\beta d^3}{\nu^2}\cdot C^*. \tag{6}$$

The gravitational acceleration is measured with reference to its value at the outer boundary g_0. The exact definition of Ra^C_{vis} depends on the choice of boundary conditions of the chemical component; that is, C^* has to be substituted by the corresponding compositional scaling.

Since the fluid is incompressible the equation of conservation of mass reduces to

$$\nabla\cdot\boldsymbol{u} = 0. \tag{7}$$

2.1. Boundary conditions

The Earth's outer core is bounded by the inner core boundary (icb) at R_i and the core-mantle boundary (cmb) at R_0. The structural and energetical evolution of the core is primarily controlled by the heat flux at the cmb because it defines the rate of secular cooling and thereby the rate of growth of the inner core (Labrosse et al., 2001; Buffett, 2003). During solidification of outer core material at the icb, latent heat is released and the mass fraction of the light chemical constituent increases in the liquid phase. This is due to the assumption that the frozen inner core comprises a lesser fraction of the light element than the surrounding liquid outer core (Poirier, 1994). Realistic boundary conditions involve time- and space-dependent flux conditions on temperature and composition at the icb. Presumably, there is virtually no transfer of composition from the outer core into the mantle so that a no flux condition would be appropriate (Anufriev et al., 2005). Nevertheless, we also consider isochemical boundary conditions at the cmb in some cases in order to demonstrate the influence of the different Prandtl numbers and the boundary conditions individually. The actual heat transfer at the cmb is likely to vary laterally as a function of the overlying convective pattern at the base of the mantle. This has been a subject of several previous studies (Zhang and Gubbins, 1996; Gibbons and Gubbins, 2000; Olson, 2003; Gibbons et al., 2007; Gubbins et al., 2011). At present, we adopt isothermal boundaries throughout for simplicity.

One aim of our study is to investigate the properties of double-diffusive core convection under the influence of different chemical boundary conditions. The first scenario considers boundaries of the Dirichlet type, i.e. fixed values of the mass fraction C. This can be expressed by

$$C|_{r=R_i} = C_0 + \Delta C \quad \text{and} \quad C|_{r=R_o} = C_0 \tag{8}$$

which are the values at the inner boundary corresponding to the icb and the outer boundary, the cmb, respectively. The total compositional difference ΔC across the shell is used for scaling when these boundary conditions are imposed. In the second scenario we fix the fluxes of C to

$$\partial_r C|_{r=R_i} = -\Phi/k_C \quad \text{and} \quad \partial_r C|_{r=R_o} = 0 \tag{9}$$

uniformly at the icb and the cmb, respectively. Here Φ denotes the dimensional flux into the shell at the icb and k_C the chemical conductivity. These are fixed chemical flux boundaries. The temporal derivative of the compositional mass fraction reference state $\dot{C}_0$