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# An innovative eigenvalue problem solver for free vibration of Euler–Bernoulli beam by using the Adomian decomposition method

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ABSTRACT

This paper deals with free vibration problems of Euler–Bernoulli beam under various supporting conditions. The technique we have used is based on applying the Adomian decomposition method (ADM) to our vibration problems. Doing some simple mathematical operations on the method, we can obtain *i*th natural frequencies and mode shapes one at a time. The computed results agree well with those analytical and numerical results given in the literature. These results indicate that the present analysis is accurate, and provides a unified and systematic procedure which is simple and more straightforward than the other modal analysis.



## 1. Introduction

The vibration problems of uniform Euler–Bernoulli beams can be solved by analytical or approximate approaches [1–4]. Using analytical approaches, the closed form solutions of free vibration under various boundary conditions have been found in these literature. Approximate approaches such as the Rayleigh–Ritz method and the Galerkin method have been applied to calculate some lower natural frequencies and mode shapes. However, it may be difficult to determine higher natural frequencies and mode shapes on account of not choosing complete and correct admissible functions. Register [5] derived a general expression for the modal frequencies and investigated the eigenvalue for a beam with symmetric spring boundary conditions. Wang [6] studied the dynamic analysis of generally supported beam using Fourier series. Yieh [7] studied the applications of dual MRM for determining the natural frequencies and natural modes of the Euler–Bernoulli beam using the singular value decomposition method. Recently, Kim [8] studied the vibration of uniform beams with generally restrained boundary conditions using Fourier series. Naguleswaran [9] obtained an approximate solution to the transverse vibration of the uniform Euler–Bernoulli beam under linearly varying axial force.

In this study, a new computed approach called Adomian decomposition method (ADM) is introduced to solve the vibration problems. The concept of ADM was first proposed by Adomian and was applied to solve linear and nonlinear initial/boundary-value problems in physics [10]. In recent years, a large amount of literature developed concerning the ADM by applying it to the applications in applied sciences. For more details about the method, see [11–15] and the references cited there. In this paper the vibration problems of uniform beams with various boundary conditions are considered. Using the ADM, the governing differential equation becomes a recursive algebraic equation and boundary conditions become simple algebraic frequency equations which are suitable for symbolic computation. Moreover, after some simple algebraic operations on these frequency equations, any *i*th natural frequency and the closed form series solution of any *i*th mode shape

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can be obtained. Finally, three problems of uniform beams are solved to verify the accuracy and efficiency of the present method.

## 2. The principle of ADM

In order to solve vibration problems by the Adomian decomposition method (ADM) the basic theory is stated in brief in this section. Consider the equation

$$Fy(x) = g(x), \tag{1}$$

where $F$ represents a general nonlinear ordinary differential operator involving both linear and nonlinear parts, and $g(x)$ is a given function. The linear terms in $Fy$ are decomposed into $Ly + Ry$, where $L$ is an invertible operator, which is taken as the highest-order derivative and $R$ is the remainder of the linear operator. Thus, Eq. (1) can be written as

$$Ly + Ry + Ny = g(x), \tag{2}$$

where $Ny$ represents the nonlinear terms in $Fy$. Eq. (2) corresponds to an initial-value problem or a boundary-value problem. Solving for $Ly$, one can obtain

$$y = \Phi + L^{-1}g - L^{-1}Ry - L^{-1}Ny, \tag{3}$$

where $\Phi$ is an integration constant, and $L\Phi = 0$ is satisfied. Corresponding to an initial-value problem, the operator $L^{-1}$ may be regarded as a definite integration from 0 to $x$. In order to solve Eq. (3) by the ADM, we decompose $y$ into the infinite sum of series

$$y = \sum_{k=0}^{\infty} y_k, \tag{4}$$

and the nonlinear term $Ny = f(y)$ is decomposed as

$$Ny = f(y) = \sum_{k=0}^{\infty} A_k, \tag{5}$$

where the $A_k$ are known as Adomian polynomials. Following [10–14], Adomian polynomials can be derived as follows:

$$\begin{aligned}
A_0 &= f(y_0),\\
A_1 &= y_1 f'(y_0),\\
A_2 &= y_2 f'(y_0) + \frac{y_1^2}{2!} f''(y_0),\\
A_3 &= y_3 f'(y_0) + y_1 y_2 f''(y_0) + \frac{y_1^3}{3!} f'''(y_0),
\end{aligned}$$

and other polynomials can be generated in a similar manner.

Plugging Eqs. (4) and (5) into Eq. (3) gives

$$\sum_{k=0}^{\infty} y_k = \Phi + L^{-1}g - L^{-1}R\sum_{k=0}^{\infty} y_k - L^{-1}\sum_{k=0}^{\infty} A_k. \tag{6}$$

Each term in series (6) is given by the recurrent relation

$$\begin{aligned}
y_0 &= \Phi + L^{-1}g,\\
y_k &= -L^{-1}Ry_{k-1} - L^{-1}A_{k-1}, \quad k \geq 1.
\end{aligned} \tag{7}$$

The initial term $y_0$ was defined and the remainder terms were determined by using simple integrations. However, in practice all terms in series (6) cannot be determined exactly, and the solutions can only be approximated by a truncated series $\sum_{k=0}^{n-1} y_k$.

## 3. Using the ADM to analyze the free vibration problem of uniform beam

Consider a uniform Euler–Bernoulli beam of finite length $l$, the equation of motion for lateral vibrations of a uniform elastic beam ignoring shear deformation and rotary inertia effects is

$$EI\frac{\partial^4 y(x,t)}{\partial^4 x} + \rho A\frac{\partial^2 y(x,t)}{\partial^2 t} = 0, \quad 0 < x < l \tag{8}$$