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On the rise of strongly tilted mantle plume tails

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ABSTRACT

The rise of an initially horizontal, buoyant cylinder of fluid through a denser fluid at low Reynolds number is used to look at the ascent of strongly tilted mantle plumes through the mantle. Such ascents are characterized by (1) the growth of instabilities and (2) the development of a thermal wake downstream. Three-dimensional numerical experiments were carried out to examine these features. An hybrid particle-in-cell finite element method was used to look at the rise of non-diffusing cylinders and, a standard finite element method was used to look at the diffusing case. First the experiments show that the timescale of the fastest growing instability vary with the Rayleigh number and the viscosity ratio. In particular the growth rate decreases as the Rayleigh number decreases, in agreement with our analysis of the laboratory experiments of Kerr et al. (2008). Second the experiments show that the length of the thermal wake increases with the Rayleigh number but the change in viscosity has almost no influence on the wake length. Applied to strongly tilted mantle plumes we conclude that such plumes cannot be unstable given the plume timescales. We also discuss the application of this conclusion to weakly tilted plumes. Besides, this study allows to predict that mantle plumes are unlikely to have developed a significant thermal wake by the time they reach the surface. Finally, the resolution that is required to allow for the growth of mantle plume tails by combined diffusion and thermal entrainment is shown to represent a challenge for the large scale mantle convection simulations.

1. Introduction

Thermal plumes are an intrinsic feature of convection. Their long-lived shape can loosely be described by a cylindrical conduit. Provided that they are not subjected to a shearing current, plumes will rise opposite to gravity, whereas, in the presence of a velocity shear, they will tilt.

The idea that plumes are deflected from the vertical has especially been portrayed in the case of the Earth's mantle convection because of the large scale plate-driven shear flow (Steinberger and OConnell, 1998). In particular, laboratory experiments (Skilbeck and Whitehead, 1978; Whitehead, 1982; Olson and Singer, 1985; Kerr and Lister, 1988) showed that gravitational instabilities of compositional plume conduits could occur if the conduits were either horizontal or tilted by a background shear by more than 60° from the vertical. In the laboratory experiments by Kerr and Lister (1988), the diffusion was compositional and not thermal, which resulted in slight diffusion only. The nature of these gravitational instabilities were analyzed by Lister and Kerr (1989) in a linear stability analysis of a buoyant cylinder of non-diffusing fluids. In particular, the authors found that a cylinder of radius a is gravitationally unstable, with the most unstable wavelength λ given by $\lambda \approx \bar{\lambda}_\infty a$, with $\bar{\lambda}_\infty = 6.28$ when the viscosity contrast $\mathcal{M} = 1$ and $\bar{\lambda}_\infty = 8.09$ when $\mathcal{M} > \sim 10$. The authors further proposed that the timescale of the fastest growing mode τ was given by $\tau \approx c_\infty(\eta_0/\Delta\rho g a)$, where η_0 is the outer viscosity, $\Delta\rho$ is the density difference between the outer fluid and the fluid within the cylindrical region, g is the gravitational acceleration and c_∞ is a constant, which equals 9.35 when $\mathcal{M} = 1$ and converges to a limit value of 2.92 as $\mathcal{M} > \sim 10$ (see Fig. 9 of Lister and Kerr, 1989). Recently, the effect of thermal diffusion on the gravitational stability of a rising horizontal cylindrical region of buoyant viscous fluid was examined by Kerr et al. (2008). At large viscosity ratios, the authors found that the instability is unaffected by diffusion when the thermal Rayleigh number $Ra = \Delta\rho g a^3/\kappa\eta_0$, defined by the temperature-driven density contrast $\Delta\rho$ between the warm cylindrical region and cool ambient fluid, the thermal diffusivity κ and the viscosity η_0 of the ambient fluid, is greater than about 300. When Ra is less than 300, diffusion significantly increases the time for instability, as the rising fluid region needs to grow substantially

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by both diffusion and entrainment before it becomes unstable. For instance, when *Ra* was less than about 140 and within an available rise height of about 40 times the cylinder radius, the rising region of fluid was unable to grow sufficiently and the instability was not seen. In particular a dimensionless time for gravitational instability at which the amplitude of the gravitational instabilities had grown to be equal to the cylinder diameter was measured. This characteristic time for instability was shown to increase at low Rayleigh numbers (Fig. 4 of Kerr et al., 2008) and suggested a trend for a decrease in growth rates at low Rayleigh numbers. Besides gravitational instabilities, Kerr et al. (2008) showed that a thermal wake left behind the buoyant cylinder develops during the rise. This downstream loss of buoyancy during a viscous rise of a thermally buoyant body was first predicted by Whittaker and Lister (2008).

Applied to the Earth's mantle, the study of Kerr et al. (2008) presented two predictions for strongly mantle plume tails. First, strongly tilted plume tails were to be stable thanks to diffusion in both the upper and lower mantle. This conclusion was essentially drawn from the low range of Rayleigh numbers *Ra* that was associated with each plume ($4.8 \leq Ra \leq 104$). Importantly this range relied on estimates of the plume radius *a* that was derived from a pipe flow model in which the plume buoyancy flux was given by models of swell uplift of the lithosphere (Davies, 1988; Sleep, 1990). Second Kerr et al. (2008) predicted the loss of some of the plume buoyancy flux from the tail in a thermal wake during the rise. This prediction implicitly meant that estimates of plume buoyancy fluxes based on mantle swells at the surface were likely to represent underestimated values of the plume flux.

The objective of this paper was to further investigate the rise of strongly tilted thermal plumes with an emphasis on the timescales for the growth of the instability and the development of the thermal wake. First we looked at the growth of the instabilities through a numerical study with experiments considering the case of the rise of an initially horizontal cylinder in a three-dimensional domain. A key to this work was to work in a 3D domain because the features of the Rayleigh–Taylor instability to be investigated were intrinsically three-dimensional. We thus studied the growth rates using two types of numerical experiments depending on whether we considered non-diffusing or diffusing cylinders (i.e. infinite and finite *Ra*, respectively). The rise of non-diffusing cylinders was simulated using a hybrid particle-in-cell finite element method while simulations of the diffusing cylinders were simulated via a standard finite element method. We also analyzed the growth rates of the instabilities for 17 of the laboratory experiments by Kerr et al. (2008) for comparison. Second we looked at the development of the thermal wake during the rise. Implications for both the stability and loss of buoyancy of strongly tilted mantle plumes were further analyzed in view of the mantle timescales. In particular we will show that the timescales provide strong evidence of (1) the stability of strongly tilted mantle plumes and (2) the unfeasibility of mantle plumes to have significantly lost their buoyancy by the time they reach the surface. We also discuss the application of this conclusion to weakly tilted plumes.

In Section 2, we describe the numerical problem. Our numerical experiments at infinite Rayleigh number are presented in Section 3, while we detail the experiments at finite Rayleigh numbers in Section 4. We present the characterization of the growth rate for the gravitational instabilities that were obtained from the laboratory experiments of Kerr et al. (2008) in Section 5. Further discussion is given in Section 6. Section 7 discusses the implications for mantle plumes. Section 8 draw the conclusions. A dimensional analysis in the finite domain is presented in Appendix A. Appendix B details the methods of analysis of the growth rate. The thermal boundary layer is finally discussed in Appendix C.

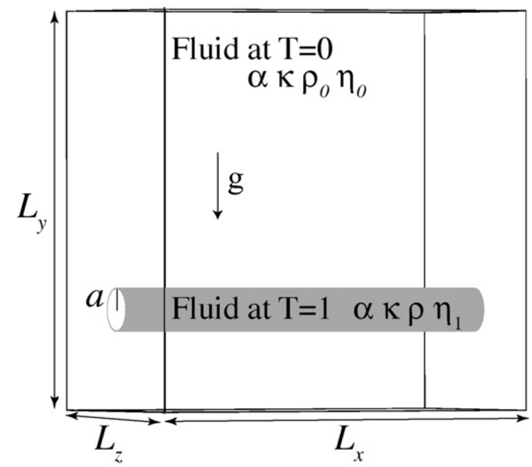

Fig. 1. Schematic of the numerical model.

2. Numerical problem description

We consider a cylinder of fluid of radius *a*, that is initially laid out horizontally in a fluid delimited by a box (Fig. 1). The fluid within the cylindrical domain has an initial temperature $T = 1$, while the outer fluid is at temperature $T = 0$. We assume that the density ρ within the box follows

$$\rho = \rho_0(1 - \alpha T), \quad (1)$$

where ρ_0 is the initial density of the fluid outside the cylinder and α is the coefficient of thermal expansion. Driven by the density difference $\Delta\rho = \rho_0 - \rho$, which initially is $\Delta\rho_0 = \alpha\rho_0$, the cylinder rises with a vertical velocity that is proportional to

$$U = \frac{g\Delta\rho a^2}{\eta_0}. \quad (2)$$

We assume that the viscosities of the outer fluid η_0 and the cylinder inner fluid η_1 are either constant or following an Arrhenius rheology (Reese et al., 1999):

$$\eta(T) = \eta_0^* \exp\left(\frac{E}{T+1}\right). \quad (3)$$

The parameters E and η_0^* were chosen so that $\eta(0) = 1$ and $\eta(1) = 1/\mathcal{M}$. We note that the total buoyancy per unit length of the cylinder is $B = \Delta\rho g a^2$.

The system is fully described by Stokes equations for incompressible creeping flow and the thermal energy equation. When the fluid properties are non-dimensionalized with respect to those of the outer fluid ρ_0 and η_0 and κ, the lengths with respect to the radius of the cylinder and time with respect to $\eta_0/\Delta\rho_0 g a$, those equations take the non-dimensionalized form:

$$-\nabla(\eta D) + \nabla p = T\hat{n}, \quad (4)$$

$$\nabla \cdot u = 0, \quad (5)$$

where $D_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ is the dimensionless strain rate tensor with dimensionless velocity u, p is the dimensionless dynamic pressure, and $\hat{n} = \hat{g}/g$ is a unit vector with $\hat{g}$ the gravitational acceleration vector, and

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \frac{1}{Ra}\nabla^2 T. \quad (6)$$

The system is found to be described by two dimensionless numbers that are the Rayleigh number

$$Ra = \frac{\Delta\rho_0 g a^3}{\kappa\eta_0}, \quad (7)$$