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Stability of sheared magnetic field in dependence on its critical level position

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ABSTRACT

A horizontal layer rotating about a vertical axis $\mathbf{z}$ at angular velocity $\mathbf{\Omega} = \Omega_0\hat{\mathbf{z}}$ and heated from below by an adverse temperature gradient $\boldsymbol{\beta} = (\mathrm{d}T/\mathrm{d}z)\hat{\mathbf{z}}$, T being the temperature, is investigated. The layer filled by inviscid Boussinesq fluid of finite conductivity between the boundaries at $z = \pm d/2$ is under a gravitational field oriented vertically downwards, $\mathbf{g} = -g\hat{\mathbf{z}}$, and permeated by a horizontal right-lined sheared magnetic field $\mathbf{B}_0$. The finite electrical conductivity is a destabilizing factor in such a system and resistive instabilities may rise under certain conditions. They are usually associated with the presence of a critical level defined by the condition $\mathbf{k} \cdot \mathbf{B}_0 = 0$, where $\mathbf{k}$ is the wave vector of perturbation and $\mathbf{B}_0$ is the ambient magnetic field. This condition is automatically satisfied by the fields featuring zero point(s). In this work the imposed magnetic field of the form $\mathbf{B}_0 = \mathcal{B}_0 \tanh \gamma(z - z_0)\hat{\mathbf{y}}$ is considered, where $\mathcal{B}_0$ is the magnitude of the field and z_0 measures its asymmetry with respect to the central plane $z = 0$. We examine the effect of varying z_0, the position of the critical level within the layer, on the stability of the system. The model is interesting from the geophysical and astrophysical point of view because of presence of an internal shear magnetic layer near the critical level associated with current sheet. The shear layer is controlled by the parameter γ which enables to change the sharpness of the magnetic field gradient across the shear layer and thereby it influences its thickness. For sufficiently large gradient of the field, the shear layer is found to be a source of hydromagnetic resistive instabilities and it is called the critical layer. A shift in the critical level location from the mid-plane towards a boundary causes the system to change in terms of the basic magnetic field (specifically its overall gradient). Boundaries were chosen to be either both perfectly conducting, insulating or a combination thereof. Linear stability analysis was performed. In general, the most preferred instabilities were found to take the form of stationary rolls. Sufficiently large gradient of the ambient magnetic field may provide conditions for exciting two distinct modes of instabilities, the *whole-layer mode* depending on global current distribution with its convective rolls extending throughout the whole layer, and the *critical-layer mode* confined to the region of the critical layer. It was found that for the layer with different electrical properties of the boundaries, the crucial importance for the onset of a particular mode (and its convective pattern) had not only the shift itself but also the fact to which boundary it was carried. Unlike the previous studies of the purely antisymmetric basic-state fields of linear and tanh profile (Tucker and Jones, 1997), and of the asymmetric field of linear profile (Marsenić and Ševčík, 2008) where only the stationary motions developed throughout the layer were possible, the presented model allows for qualitatively different modes localized mainly around the critical level. The confinement of the convection to the region of the critical layer was possible just for a sufficiently steep shear of the magnetic field and for the thin shear layer being shifted to the perfectly conducting boundary.

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1. Introduction

Studies of magnetohydrodynamic (MHD) systems, where cosmic magnetic fields are generated, often neglect omnipresent electrical resistivity, which usually is very small. Change of the field topology is not possible then, magnetic field is "frozen" to the perfectly electrically conducting fluid. Eventual instabilities of such (inviscid) systems are ideal, nurtured by the magnetic field gradients when large magnetic stresses cause the material to be thrown about. For that reason they are called also the gradient instabilities and the necessary condition for their rise is the magnetic field lines to be curvilinear. Finite resistivity may destabilize the system predicted as stable in the perfectly conducting limit. The most important resistive effect is reconnection; change of the magnetic field lines' topology due to local diffusive processes operating on

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small space-scales. The reconnecting magnetic field configuration occurs wherever the field veers its direction fast enough to make the local processes overtake the global decay of the magnetic field.

The geomagnetic field is a result of complicated MHD processes occurring within the liquid, electrically conducting, stratified Earth's outer core. Stratification of the fluid results from its thermal and compositional inhomogeneity and it is considered to be the main source for dynamo and the field variations. At certain circumstances, mechanisms at which the fluid stratification is unimportant but the instability is driven at the expense of magnetic energy may be crucial. In order to identify the instability mechanism responsible for magnetic field variations, one should look on their time-scales which are significantly affected by rotation as compared with non-rotating cases. In rotating systems, the ideal instabilities originate from the balance of dynamical Coriolis and Lorentz forces and the time τ_s at which they occur is given by

$$\tau_s = \frac{2\Omega_0 H^2 \mu\rho}{\mathcal{B}_0^2}, \tag{1}$$

where μ and ρ is the magnetic permeability and density of the fluid, H is the typical length-scale of the system, Ω_0 is the rotation rate and $\mathcal{B}_0$ is the typical strength of the magnetic field. For parameters of the Earth's core $\tau_s = O(10)$ yr. On the other hand, the magnetic diffusion time τ_η, associated with the geomagnetic field decay, posing the upper time-scale limit for possible hydromagnetic processes

$$\tau_\eta = \frac{H^2}{\eta}, \tag{2}$$

where $\eta = 1/\mu\sigma$ is the diffusivity of the fluid and σ is its conductivity, is of order $O(10^4)$ yr. More on hydromagnetic processes and their time-scales can be found for example in Jacobs (1987). The ratio of these time-scales gives an important dimensionless parameter called Elsasser number

$$\Lambda = \frac{\tau_\eta}{\tau_s} = \frac{\sigma \mathcal{B}_0^2}{2\Omega_0 \rho}, \tag{3}$$

which can be viewed as the measure of electrical conductivity or, alternatively, as the measure of magnetic field strength.

Resistive instabilities represent a potentiality as a source for magnetic field variations. They have been originally studied in the classical non-rotating magnetohydrodynamics in order to obtain the stability conditions for the magnetic fields used in fusion research. Magnetostatic equilibrium is the result of balance between the mechanical forces and magnetic stresses. The dynamic non-dissipative time-scale in the non-rotating case is the Alfvén time-scale τ_A expressed by

$$\tau_A = \frac{H\sqrt{\mu\rho}}{\mathcal{B}_0}. \tag{4}$$

The study Furth et al. (1963) gave a theoretical survey of resistive instabilities in a compressible, density stratified plasma forming a sheet pinch. From among the unstable modes detected, the magnetic tearing mode is probably the most relevant for usually considered conditions within Earth's core. It does not require compressibility of the plasma. The authors described it as a purely growing mode which is symmetric about the mid-plane of the sheet pinch. It is rather a long-wavelength than short-wavelength mode relative to the dimension of the current layer between two differently directed magnetic fields pressed together, see also Cowling (1976), Sturrock (1994) or Priest (1982). It occurs at the time-scale which satisfies

$$\tau_A < t_{tearing} < \tau_\eta. \tag{5}$$

Tearing-mode instability thus represents the process fast enough to outrun the global magnetic field diffusion and is important for its potentiality to change the magnetic field topology. In rotating plasmas, Coriolis force modifies the tearing instability time-scale

$$\tau_s < t_{tearing} < \tau_\eta. \tag{6}$$

but its principal mechanism remains the same like in the non-rotating case. For finite fluid conductivity, the mechanical force $\mathbf{j} \times \mathbf{B}_0$, $\mathbf{j}$ being the electric current, prevents motions transverse to the magnetic field lines, see for example Cowling (1976). However, it becomes very weak near the points where the magnetic field vanishes, what is the key factor for the motion to rise. This situation is expressed by the condition

$$\mathbf{k} \cdot \mathbf{B}_0 = 0, \tag{7}$$

where $\mathbf{k}$ is the wave vector of a perturbation. The level at which the condition (7) is satisfied is called the critical level. Depending on the magnetic field gradient, a thin transition layer may develop around the critical level. This critical layer can be a source of resistive tearing mode, which leads to reconnection of the field lines at a faster rate than the Ohmic diffusion permits. The authors in the works Kuang and Roberts (1990) and Kuang and Roberts (1991) studied the stability of the sheet pinch created by a conducting fluid layer between the perfectly conducting walls at $z = 0, d$ and rotating about a vertical $\mathbf{z}$. The horizontal sheared field in the form

$$\mathbf{B}_0 = \mathcal{B}_0 \left[\hat{\mathbf{x}} \cos\left(\frac{qz}{d}\right) + \hat{\mathbf{y}} \sin\left(\frac{qz}{d}\right) \right] \tag{8}$$

was considered. The parameter q was the measure of the field twist and also determined the number of critical levels. For $q > \pi$ at least one critical level is secured inside the layer and stability of the homogeneous fluid is lost when Λ exceeds a critical value Λ_c. It was shown that the tearing mode is significantly suppressed by the rotation as compared with the classical case and the most favourable conditions for its growth set for $\Lambda = O(1)$. The second mentioned study was devoted to resistive instabilities including the effect of gravity, measured by the Rayleigh number R. Conditions for the marginal instability were referred to the value R_c, the value when the ideal stability is lost. Attention was concentrated on resistive g-modes, which can exist in the system when critical levels are absent and they grow at the rate typical for Ohmic evolution of the field. Presence of a critical level helps to their growth. Kuang (1994) examined the effect of boundary conditions on the stability of homogeneous rotating fluid permeated by the magnetic field (8) in the high-conductivity limit. It was found that the system with one wall insulating provides conditions for enhancing the tearing instability. The author suggested that the enhancement was due to diffusive processes on the insulating wall. Furthermore, the insulating boundary allows for instability to occur even if there is not any critical level. The instability then draws energy from the diffusive processes in the resistive layer at the boundary and so it is referred to as a boundary mode. Critical levels, if any, behave like impenetrable perfectly conducting walls to this mode, so the boundary mode is confined between the insulating boundary and the critical level.

The presented work is motivated by the result of the numerical simulation of the geomagnetic field evolution in Glatzmaier et al. (1995) or Glatzmaier and Roberts (1999). The obtained magnetic field patterns show the feature necessary for the tearing instability to occur, namely a steep shear of the toroidal field which is localized in the core polar regions and near the conducting inner-core boundary. This feature is present in all stages of the field evolution, i.e. before the reversal, during it and after the reversal. It suggests that possible resistive instability may be relevant and important process in Earth's core. The horizontal-layer model with an asymmetric shear ambient field seems to be adequate to explore the role of particular attributes of the system; the critical level location, the field gradient largeness and the boundary conditions. This study extends the work Tucker and Jones (1997), which examined

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