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Multiple inner core wobbles in a simple Earth model with inviscid core

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ABSTRACT

The inner core wobble (ICW) is the chandler wobble of the inner core. Its predicted period for the PREM model is about 7.5 years, based upon the resolution of the Liouville equations of conservation of angular momentum. Here, solving the local equation of conservation of linear momentum with a truncated chain that couples the toroidal and spheroidal displacement fields, the ICW is computed for a model made up of three homogeneous layers: an incompressible liquid outer core and rigid mantle and inner core. Contrary to the angular momentum approach, as implemented up to now, that provides a single ICW, the linear momentum approach shows that the dynamics of the neutrally stratified outer core may generate a family of ICWs with periods ranging from a few dozens to thousands of days. The mode with the largest wobble amplitude in the inner core has a period close to that obtained with the angular momentum approach.

1. Introduction

The volume, mass, and mean moment of inertia of the solid inner core are, respectively, 0.7, 1.7, and 0.07% of those of the whole Earth. Because these quantities are so small and the inner core is surrounded by the fluid outer core, it weakly couples to the mantle and, at the surface, some of its motions are barely or not yet observable by present techniques. The coupling to the outer core and mantle may be, for instance, mechanical, topographic, gravitational, electromagnetic, or viscous. Examples of free motions almost entirely confined to the inner core are its translational free oscillations, also called the Slichter modes, the free inner core nutation (FICN) and inner core wobble (ICW), which is its chandler wobble (CW). This paper is devoted to the theoretical calculation of the ICW. It is one of the four rotational modes of a rotating ellipsoidal Earth model, the others being the free core nutation (FCN), FICN, and CW. Like the Slichter modes, the ICW heavily depends on the difference between the mean density of the inner core and the density of the outer core at the inner core–outer core boundary (ICB). It was first theoretically considered by Busse (1970) to explain the Markowitz wobble (Markowitz, 1960, 1968) which is a decadal polar motion that is currently still unexplained (Dumberry, 2008). Noticeable improvements to Busse's model were brought by Kakuta et al. (1975), Mathews et al. (1991a), Rochester and Crossley (2009) and Dumberry (2009). In these studies, the rotational modes are computed by solving the equations of conservation of angular momentum, or Liouville's equations, which are obtained by integrating over the inner core, outer core, and entire Earth the cross product of the position vector and the local equations of motion. The elastic deformation and outer core flow accompanying the rigid nutation are computed for a non-rotating spherical model perturbed by a static body force that is the variation of the centrifugal force induced by the rigid nutation. The angular momentum approach is therefore a hybrid method involving both the local equations of conservation of linear momentum and volume integrals.

From the observational point of view, the search by Guo et al. (2005) of a signal related to the ICW in polar motion data was unsuccessful.

In this paper, I use the normal modes theory of a rotating ellipsoidal Earth model (Smith, 1974) to investigate the ICW of a model made up of three homogeneous layers: rigid inner core and mantle and an incompressible fluid outer core. The method consists in numerically solving a truncated set of ordinary differential equations over radius obtained by expanding the equations of conservation of linear momentum in generalized spherical harmonics. Its main advantage over the angular momentum approach is that it includes the seismic normal modes, the inertia-gravity spectrum of the liquid core, and the rotational modes. In particular, it takes account of the interaction between the rotational modes and the inertia-gravity spectrum of the core, whereas the angular momentum approach is meant for studying the rotational motions only.

The paper is organized as follows. In Section 2, the major theoretical approaches to the calculation of the ICW are briefly reviewed, as well as the significant results derived from them. Section 3 contains a short description of the linear momentum description adopted here to compute the ICW. In Section 4,

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Table 1
Earth model made up of three uniform layers. Rigidity in both the inner core and mantle is approached by taking a large S-wave velocity and incompressibility is approached by taking a large P-wave velocity. The wave velocities are 10^3 to 10^4 bigger than those of realistic Earth models.

Layer	Radius (km)	ρ (kg/m^3)	v_p (m/s)	v_s (m/s)
Inner core	1221.5	12,894	5×10^7	5×10^6
Outer core	3480.0	10,901	5×10^7	0
Mantle	6371.0	4,449	5×10^7	5×10^6

numerical results for the ICW of the simple model mentioned above are reported on. Section 5 will complete the paper with concluding remarks.

2. Busse's, Kakuta et al.'s and angular momentum approaches

Busse (1970) considered an ellipsoidal Earth model that is rotating at a mean angular speed Ω and made up of three homogeneous layers: a rigid inner core, an incompressible liquid outer core, and a rigid mantle. By neglecting gravitation, he obtained the following analytical formula for the ICW eigenfrequency:

$$\omega_{\mathrm{ICW}} = \frac{\rho_{\mathrm{IC}} - \rho_{\mathrm{OC}}}{\rho_{\mathrm{IC}}} e_{\mathrm{IC}} \Omega, \tag{1}$$

where ρ_{IC} and ρ_{OC} are the densities of the inner and outer cores, respectively,

$$e_{\mathrm{IC}} = \frac{C_{\mathrm{IC}} - A_{\mathrm{IC}}}{A_{\mathrm{IC}}}, \tag{2}$$

is the dynamical flattening and A_{IC} and C_{IC} are, respectively, the equatorial and polar moments of inertia of the inner core. Besides, he assumed that the flow in the outer core is geostrophic and, as a consequence of the Taylor–Proudman theorem, is confined inside a cylinder parallel to the rotation axis and tangent to the ICB.

Kakuta et al. (1975), who, contrary to Busse (1970), included gravitation in the local equation of conservation of linear momentum for the outer core, also obtained the ICW eigenfrequency (1). But, as was pointed out by Rochester and Crossley (2009), this resulted from a small error in the estimation of the gravitational torque exerted on the inner core. The correct eigenfrequency is actually given by formula (3).

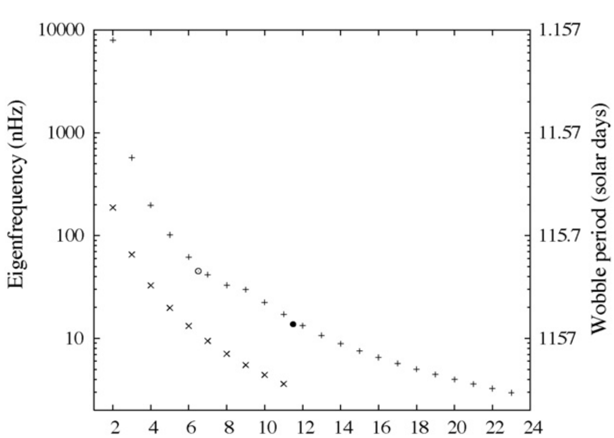

Fig. 1. Eigenfrequencies of CW, ICW, and pseudo-modes of the Earth model described in Table 1 as a function of the number of nodes of $W_1^{-1}(r)$ in the outer core. They are computed for the TST coupling chain (7), except for the ICW eigenfrequency given by Eq. (3), which is designated by a filled circle. The CW is designated by an empty circle. The positions of both the CW and ICW on the horizontal axis are arbitrary. Most of the energy of the pseudo-modes marked by a + (resp. ×) sign is concentrated in the lower (resp. upper) outer core, as shown in Fig. 2 (resp. Fig. 3).

Table 2
For the Earth model described in Table 1, ICW period (in solar days) corresponding to eigenfrequency (1) (Busse, 1970), CW and ICW periods (in solar days) obtained by using the angular momentum (AM) approach (Mathews et al., 1991a; Rochester and Crossley, 2009) and linear momentum (LM) approach with a TST coupling chain Eq. (7). In the AM approach, the ICW eigenfrequency is given by formula (3).

	CW	ICW
Eq. (1)	–	2583
AM	256	842
LM	256	Multiple (see Fig. 2)

Mathews et al. (1991a) considered a more elaborate Earth model. The layers are heterogeneous, the fluid outer core is compressible, and the mantle is elastic. But, elastic deformations of the inner core in response to its tilt were not properly taken into account when calculating the ICW. Their theoretical approach relied on the Liouville equations of conservation of angular momentum for the inner core, outer core, and whole Earth, and on the coupling between the layers owing to pressure and gravitational torques at both the ICB and CMB. They derived an expression for the ICW eigenfrequency that yields formula (1) when the same model as that of Busse (1970) is considered and gravitational coupling between the inner core and mantle is neglected:

$$\omega_{\mathrm{ICW}} = \alpha_3 (1 + \alpha_g) e_{\mathrm{IC}} \Omega, \tag{3}$$

where

$$\alpha_3 = 1 - \frac{A'e'}{A_{\mathrm{IC}} e_{\mathrm{IC}}}, \tag{4}$$

$$\alpha_g = \left(\frac{3G}{a_{\mathrm{IC}}^5 \Omega^2} \right) \left[\left(1 + \frac{5\bar{\rho}_{\mathrm{IC}}}{3\rho_{\mathrm{OC}}(a_{\mathrm{IC}})} \right) A'e' - A_{\mathrm{IC}} e_{\mathrm{IC}} \right] - 1, \tag{5}$$

A' and e' are, respectively, the equatorial moment of inertia and dynamical flattening of an ellipsoid the same mean radius a_{IC} as the inner core and constant density equal to the density $\rho_{\mathrm{OC}}(a_{\mathrm{IC}})$ of the outer core at the ICB, and $\bar{\rho}_{\mathrm{IC}}$ is the mean density of the inner core. Notably, the outer core compressibility does not enter Eq. (3) and gravitational coupling between the inner core and the rest of the Earth increases the eigenfrequency given by Eq. (1) by a factor 3 or 4.

Rochester and Crossley (2009) showed that elasticity of the inner core increases the ICW period by 10–15%, depending on the Earth model. They, like Mathews et al. (1991a), solved the Liouville equations for the conservation of angular momentum. The novelty of their method resides in the Lagrangean formulation of the Liouville equations. Moreover, they found that, at very long periods, solving Poisson's equation in the outer core does not require a detailed knowledge of the displacement field in the outer core. The period they obtain for the PREM model (Dziewonski and Anderson, 1981) is approximately 2750 days. Stimulated by Rochester and Crossley's (2009) paper, Dumberry (2009) reworked the analysis of Mathews et al. (1991a,b) to take fully into account the elastic deformations of the inner core and arrived at numerical results similar to those of Rochester and Crossley (2009).

In Section 4, comparison will be made between the ICW eigenfrequencies (1) and (3) and those provided by the numerical resolution of the equations of conservation of linear momentum.

3. Linear momentum approach

The results presented in this paper are based on the theory of the normal modes of a rotating self-gravitating elastic Earth model initially developed by Smith (1974) and its amendments by Schastok (1997), Rogister (2001), Rogister (2003) and Huang et al. (2004). Of course, the normal modes are the solutions of the local equation of conservation of linear momentum where the external forces are