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# Tidal instability in stellar and planetary binary systems

M. Le Bars[a,*], L. Lacaze[b], S. Le Dizès[a], P. Le Gal[a], M. Rieutord[c]

[a] *Institut de Recherche sur les Phénomènes Hors Equilibre, UMR 6594, CNRS et Aix-Marseille Université, 49 rue F. Joliot-Curie, BP146, 13384 Marseille Cédex 13, France*
[b] *Institut de Mécanique des Fluides de Toulouse, CNRS et Université de Toulouse, 1 Allée du Professeur Camille Soula, 31400 Toulouse, France*
[c] *Laboratoire d'Astrophysique de Toulouse-Tarbes, CNRS et Université de Toulouse, 14 avenue E. Belin, 31400 Toulouse, France*

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ABSTRACT

In this paper, we combine theoretical and experimental approaches to study the tidal instability in planetary liquid cores and stars. We demonstrate that numerous complex modes can be excited depending on the relative values of the orbital angular velocity $\Omega_{orbit}$ and of the spinning angular velocity $\Omega_{spin}$, except in a stable range characterized by $\Omega_{spin}/\Omega_{orbit} \in [-1;1/3]$. Even if the tidal deformation is small, its subsequent instability – coming from a resonance process – may induce motions with large amplitude, which play a fundamental role at the planetary scale. This general conclusion is illustrated in the case of Jupiter's moon Io by a coupled model of synchronization, demonstrating the importance of energy dissipation by elliptical instability.



## 1. Introduction

The fundamental role of tides in geo- and astrophysics has been the subject of multiple studies for more than four centuries. Beyond the well-known quasi-periodic flow of ocean water on our shores, tides are also responsible for phenomena as varied as the intense volcanism on Io or the synchronization of the Moon on Earth. In stars and liquid planetary cores, tides may also excite an hydrodynamic "elliptical" instability, whose consequences are not yet fully understood. The purpose of the present work is twofold: we shall first systematically characterize the excited modes of the elliptical (or tidal) instability in a rotating spheroid depending on its orbital and spinning velocities, and then demonstrate the importance of this instability in stellar and planetary binary systems using a simplified but illustrative model of tidal synchronization.

The elliptical instability, whose existence is related to a parametric resonance of inertial waves, is well-known in aeronautics, and more generally in the field of vortex dynamics: it actually affects any rotating fluid, as soon as its streamlines are elliptically deformed. Since its discovery in the mid-1970s, the elliptical instability has received considerable attention, theoretically, experimentally and numerically (see for instance the review by Kerswell, 2002). Its presence in planetary and stellar systems, elliptically deformed by gravitational tides, has been suggested for several years. It could for instance be responsible for the surprising existence of a magnetic field in Io (Kerswell and Malkus, 1998; Lacaze et al., 2006; Herreman et al., 2009) and for fluctuations in the Earth's magnetic field on a typical timescale of 10,000 years (Aldridge et al., 1997). It may also have a significant influence on the evolution of binary stars (e.g. Rieutord, 2003).

In all these studies, it is assumed that the tidal deformation is fixed and that the excited resonance is the so-called spin-over mode, which corresponds to a solid body rotation around an axis inclined compared to the spin axis of the system. This is indeed the only perfect resonance in spherical geometry in the absence of rotation of the elliptical deformation (Lacaze et al., 2004). But in all natural configurations such as binary stars, moon–planet systems or planet–star systems, orbital motions are also present, which means that the gravitational interaction responsible for the tidal deformation is rotating with an angular velocity and/or a direction different from the spin of the considered body. This significantly changes the conditions for resonance and the mode selection process, as recently demonstrated in the cylindrical geometry (Le Bars et al., 2007).

The paper is organized as follow. In Section 2, in complement to the trends presented in Le Bars et al. (2007), we systematically characterize the excited modes of the elliptical instability in a rotating spheroid depending on its orbital and spinning velocities, using both theoretical and experimental approaches. We then describe in Section 3 a fully coupled simplified model of synchronization of stellar and planetary binary systems, demonstrating the importance of energy dissipation by elliptical instability. In the last section, the main results of the paper are summarized and general conclusions for geo- and astrophysical systems are briefly discussed.

* Corresponding author. Tel.: +33 4 96 13 97 81; fax: +33 4 96 13 97 09.
*E-mail address:* lebars@irphe.univ-mrs.fr (M. Le Bars).

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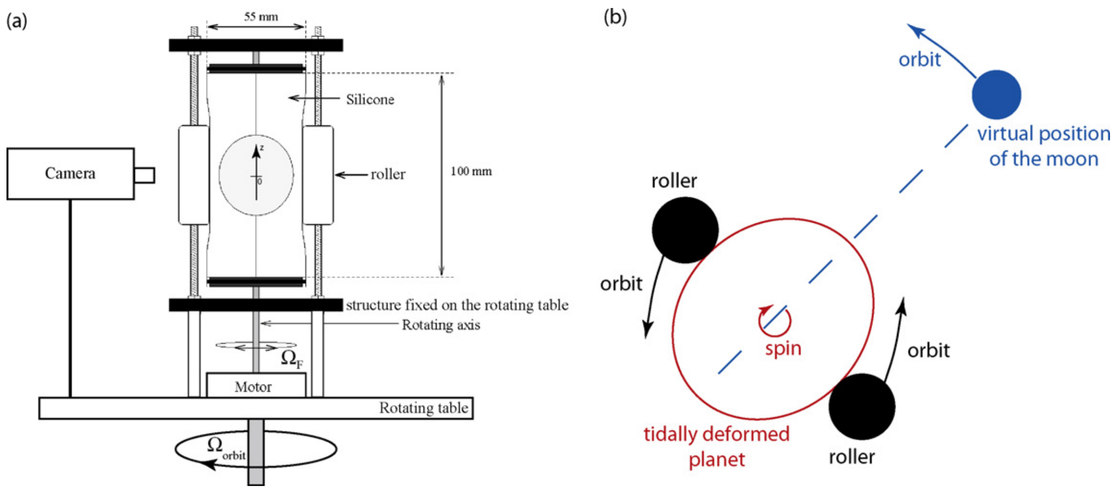


**Fig. 1.** (a) Sketch of the experimental set-up and (b) correspondence with the geophysical configuration (top view).

## 2. Excited modes of the elliptical instability in an orbiting spinning spheroid

Our study is based on the laboratory experiment shown in Fig. 1(a). The set-up consists in a deformable and transparent hollow sphere of radius $R = 2.175$ cm, set in rotation about its axis ($Oz$) with an angular velocity $\Omega_F$ up to $\pm 300$ rpm, simultaneously deformed elliptically by two fixed rollers parallel to ($Oz$). The container is filled with water seeded with anisotropic particles (Kalliroscope). For visualization, a light sheet is formed in a plane coinciding with the rotation axis, allowing the measurement of wavelengths and frequencies of excited modes. The whole set-up is placed on a 0.5 m-diameter rotating table allowing rotation with an angular velocity $\Omega_{orbit}$ up to 60 rpm. Such a system is fully defined by three dimensionless numbers: $\varepsilon$, the eccentricity of the tidal deformation, $\Omega = \Omega_{orbit}/\Omega_F$, the ratio between the orbital and the fluid angular velocities, and $E = \nu/\Omega_F R^2$, an Ekman number, where $\nu$ is the kinematic viscosity of the fluid. In geo- and astrophysical terms, this toy model mimics a tidally deformed fluid body spinning at $\Omega_{spin} = \Omega_F + \Omega_{orbit}$ with a tidal deformation rotating at the orbital velocity $\Omega_{orbit}$ (see Fig. 1(b)). Note that in natural configurations, the gravitational interactions responsible for the boundary deformation of the considered planet or star also act over the whole depth of the system. This feature cannot be taken into account in our toy model. However, it touches another side of the problem, namely the role of compressibility and stratification which we leave for subsequent studies. We focus here on incompressible effects only, considering a fluid of uniform density.

### 2.1. Linear global analysis

As previously mentioned, the elliptical instability is generated by the parametric resonance of two normal modes of the undistorted circular flow with the underlying strain field (e.g. Waleffe, 1990; Kerswell, 2002). We have thus performed a so-called "global" analysis of the instability, which consists in (i) determining the normal modes of the sphere, (ii) calculating explicitly the conditions for resonance, which immediately provides information on the structure of the selected instability and (iii) determining the growth rate of this instability. In the following, we work in the frame rotating with the rotating table (i.e. in the frame where the elliptical deformation is stationary), and variables are nondimensionalized using the characteristic lengthscale $R$ and the characteristic timescale $1/\Omega_F$ (i.e. the relevant timescale for the elliptical instability, corresponding to the rotation of the fluid compared to the elliptical deformation).

As explained in Le Bars et al. (2007), inviscid normal modes in a rotating container submitted to a global rotation $\Omega$ are related to inviscid normal modes without global rotation through the relation:

$$\{\mathbf{u}, p\}(\omega, \Omega, m, l) = \left\{ \frac{\mathbf{u}}{1+\Omega}, p \right\} (\tilde{\omega}, 0, m, l) \quad (1)$$

where $\mathbf{u}$ and $p$ stand for the velocity and the pressure, respectively. Here, $\omega$ is the mode frequency in the frame rotating with the elliptical deformation, $\tilde{\omega} = (\omega + m\Omega)/(1+\Omega)$, and $m$ and $l$ are azimuthal and "meridional" wavenumbers respectively (see Lacaze et al., 2004, for details). Due to this property, the dispersion relation solutions in the sphere with global rotation are the same as the ones given by Lacaze et al. (2004) without global rotation when $\omega$ is replaced by $\tilde{\omega}$. The linear analysis of the elliptical instability in the rotating frame can thus be expressed from the results obtained without global rotation. The condition for resonance between two waves is simply given by $(m_2,\omega_2) = (m_1+2,\omega_1)$, and the corresponding excited resonance is labeled by $(m_1,m_2)$. Note that as frequencies of normal modes are confined to the interval $m-2 < \tilde{\omega} < m+2$, resonances are only possible for $\Omega$ outside the range $[-3/2;-1/2]$. There, the growth rate $\sigma_1 = \sigma/\varepsilon$ is solution of the equation (see again Lacaze et al., 2004, for details):

$$\left(\sigma_1 \tilde{J}_{1|1} - \frac{\sqrt{E}\nu_s^1(1+\Omega)^2}{\varepsilon} - \tilde{C}_{1|1}\right)\left(\sigma_1 \tilde{J}_{2|2} - \frac{\sqrt{E}\nu_s^2(1+\Omega)^2}{\varepsilon} - \tilde{C}_{2|2}\right)$$
$$= (\tilde{N}_{1|2} - (1+\Omega)\tilde{I}_1)(\tilde{N}_{2|1} - (1+\Omega)\tilde{I}_2), \quad (2)$$

where $\tilde{J}_{i|i}$ corresponds to the norm of mode $i$, $\tilde{N}_{i|j}$ to the coupling coefficient between modes $i$ and $j$, $\nu_s^i$ to the viscous damping induced by the no-slip boundary condition on each mode derived from the work of Kudlick (1966),[1] $\tilde{I}_i$ to surface effect induced by the elliptic shape of the boundary and $\tilde{C}_{i|i}$ to the possible detuning of the instability when $\Omega$ is slightly off the perfect resonance condition. The exact expressions of all these coefficients are given in Appendix A.

Numerical resolution of Eq. (2) determines the growth rate of any given resonance depending on the dimensionless parameters ($\varepsilon$,$\Omega$,$E$). Our computations demonstrate that only principal reso-

[1] Note that only boundary layer effects are considered here, and that damping due to inner shear layers are neglected. This assumption has been fully justified by numerical computation for the spin-over mode (Hollerbach and Kerswell, 1995), and is supposed to remain valid here.