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An empirical comparison among aftershock decay models

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ABSTRACT

We compare the ability of three aftershock decay models proposed in the literature to reproduce the behavior of 24 real aftershock sequences of Southern California and Italy. In particular, we consider the Modified Omori Model (MOM), the Modified Stretched Exponential model (MSE) and the Band Limited Power Law (LPL). We show that, if the background rate is modeled properly, the MSE or the LPL reproduce the aftershock rate decay generally better than the MOM and are preferable, on the basis of the Akaike and Bayesian information criteria, for about one half of the sequences. In particular the LPL, which is usually preferable with respect to the MSE and fits well the data of most sequences, might represent a valid alternative to the MOM in real-time forecasts of aftershock probabilities. We also show that the LPL generally fits the data better than a purely empirical formula equivalent to the aftershock rate equation predicted by the rate- and state-dependent friction model. This indicates that the emergence of a negative exponential decay at long times is a general property of many aftershock sequences but also that the process of aftershock generation is not fully described by current physical models.

1. Introduction

The most commonly used formula to reproduce the decay of aftershock rate after a mainshock, also adopted in procedures for the real-time forecast of aftershock probabilities in California (Gerstenberger et al., 2007), is the Modified Omori Model (MOM, Utsu, 1961)

$$\lambda_{\rm MOM}(t) = \frac{K}{(t+c)^p} \tag{1}$$

where $\lambda_{\rm MOM}(t)$ is the intensity (the rate) of a non-stationary Poisson process, and p, c and K are free parameters. The MOM is empirical in nature but it was found to be compatible with the rate- and state-dependent friction model proposed by Dieterich (1994).

A characteristic of the MOM is to predict an infinite number of possible future aftershocks (that is an infinite number of potential faults) if the power law exponent p is lower than or equal to 1. Since such p values are often observed for real sequences, the MOM might appear physically unrealistic. Few alternative formulations, proposed in the last decades, overcome this limitation of the MOM. We mainly consider here two of them: the Modified Stretched Exponential Model (MSE, Kisslinger, 1993; Gross and Kissilinger, 1994) and the Band Limited Power Law (LPL, Narteau et al., 2002, 2003). Both MSE and LPL assume that a negative exponential decay emerges at long times, hence they predict a finite number of aftershocks (and faults), independently of the value of the power law exponent. The intensity of the MSE can be written as

$$\lambda_{\rm MSE}(t) = (1-r)N^*(0)\exp\left[\left(\frac{d}{t_0}\right)^{1-r}\right]\frac{(t+d)^{-r}}{t_0^{1-r}} \times \exp\left[-\left(\frac{(t+d)}{t_0}\right)^{1-r}\right] \tag{2}$$

where $N^*(0)$ is the total number of potential shocks at the time of the mainshock ($t=0$), t_0 is the relaxation time of the negative exponential decay process, d a delay time (corresponding to parameter c of the MOM) and $0<r\leq 1$ the power–law exponent.

The intensity of the LPL is given by

$$\lambda_{\rm LPL}(t) = B\frac{\gamma(q,\lambda_b t)-\gamma(q,\lambda_a t)}{t^q} \tag{3}$$

where q is the power–law exponent, B is a normalizing constant (similar to K of the MOM), $\lambda_a > \lambda_b$ are two parameters (having the physical dimensions of rates) that controls the behavior at long and short times respectively, and γ indicates the incomplete Gamma function

$$\gamma(q,x) = \int_0^x z^{q-1}e^{-z}dz \tag{4}$$

When $\lambda_b \gg \lambda_a$ (as it may be assumed usually) the behavior of the LPL can be described as the superposition of three regimes that control the rate at different times: an initial linear decay, which is

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followed by a power–law and, at large times, by a negative exponential. Narteau et al. (2003) suggested to considering two times $t_1(\chi) < t_2(\chi)$ that correspond to the transition between the linear and the power–law and between the power–law and the exponential decays respectively. They are defined as the times at which the ratio between aftershock rates predicted by LPL and by a pure power–law is χ. Narteau et al. (2003) report the values assumed by $t_1(\chi)$ and $t_2(\chi)$, for values of the ratio χ ranging from 0.8 to 0.99 while Lolli et al. (2009) proposed to use $t_b = t_1(2^{-q})$ and $t_a = t_2(1/e)$ (where e is the base of natural logarithms) as they corresponds approximately to c of the MOM (or d of MSE) and t_0 of the MSE respectively. We will adopt such derived parameters t_b and t_a in the following references to the LPL.

Both MSE and LPL are based on reasonable physical assumptions but Lolli and Gasperini (2006) showed that MSE and LPL are preferable with respect to the MOM for about one fourth of the real aftershock sequences of Southern California and Italy only. They hypothesized that the limited duration of the observing time interval they choose (one year) might penalize the MSE and the LPL with respect to the MOM when the exponential decay emerges later than the end of such interval. In this work we will test such hypothesis by considering a longer observing interval of four years. We also consider here the possibility that the background rate (not modeled by Lolli and Gasperini, 2006) might play a role in assigning the preference to the MOM in some cases. The background seismicity rate is accounted simply by a constant rate μ (to be determined together with the other parameters of the various decay models) added to the rate Eqs. (1)–(3).

2. Data sources and sequence detection

We use essentially the same datasets analyzed by Lolli and Gasperini (2006) but we extend the analysis to a longer time interval of four years after the mainshock and consider the catalogs of Southern California and Italy up to July 2008 and May 2008 respectively (instead of December 2004). For California we use the revised catalog from 1932 to 2008 available from the Southern California Earthquake Center (SCEC) site (http://www.scecdc.scec.org/). For Italy, we merged several catalogs of Italian instrumental earthquakes covering the time interval from 1960 to May 2008. From 1960 to 1980, we used the catalog of the Progetto Finalizzato Geodinamica (Postpischl, 1985) with magnitudes corrected according to Lolli and Gasperini (2003); from 1981 to 1996, we used the *Catalogo Strumentale dei Terremoti Italiani* dal 1981 a 1996 Version 1.1 (CSTI Working Group, 2004); from 1997 to 2002, we used the Catalogo della Sismicità Italiana 1.1 (Castello et al., 2005); finally, from 2003 to 2008, the data are taken from the instrumental bulletin of the *Istituto Nazionale di Geofisica e Vulcanologia* (INGV) available from site http://www.ingv.it/~roma/reti/rms/bollettino. Following Ouillon and Sornette (2005), we assumed the completeness of the Southern California catalog for $M_L > 3.0$ in 1932 and later years, for $M_L > 2.5$ in 1975 and later years, $M_L > 2.0$ in 1992 and later years, and $M_L > 1.5$ in 1994 and later years. For Italy we assumed the completeness for $M_L > 2.5$ for 1984 and later (Lolli and Gasperini, 2003), and for $M_L > 3.0$ before 1984.

In a first step we use the same sequence detection algorithm adopted by Lolli and Gasperini (2006) that defines the influence zone of any shock as a circular area centered in the epicenter and assumes as mainshocks (triggering the sequences) all earthquakes with magnitude not lower than 5.0 that are not included in the influence zone of a larger shock. The time window is fixed to four years after the mainshock while the radius R of the influence zone is chosen as a function of magnitude as $\mathrm{Log}\,10(R) = 0.1238\mathrm{M} + 0.983$ (that closely corresponds to Table 1 of Gardner and Knopoff, 1974). Only the shocks shallower than 40 km and with magnitude above completeness threshold are included in sequences. To reduce the possible incompleteness in the first times after the mainshock we only consider the aftershocks with magnitude not lower than mainshock magnitude M_m minus 3.5.

As the Gardner and Knopoff (1974) radius is likely to overestimate the size of the mainshock influence zone, in a second step we performed an analysis of correlation between the shock rates observed at different distances from the mainshock during a time interval of 200 days after the mainshock. In particular, for distances r varying from 0 to R, we correlate the sequence of rates observed (over 5 days bins) inside the circle with radius r and inside the circular ring with minimum and maximum radius r and R respectively. For each sequence we assumed as influence distance (reported in Table 1 as R_i) the largest r for which the correlation between the sequences of rates is significant at the 0.05 level.

To grant a reliable determination of model parameters we consider for the analysis only the sequences including 100 shocks at least within the four years time interval following the mainshock. Moreover, since all the simple decay models we consider are not suitable to reproduce complex sequences with strong secondary clustering we excluded from our dataset, by a visual analysis of the plot of the rate over 5 days bins, the sequences showing at later times one or more peaks of the shock rate with amplitude of the same order of magnitude of the peak following the mainshock.

The detected sequences are listed in Table 1. The longer time window (four years instead of one) and the different completeness thresholds and selection criteria here adopted reduces the number of sequences (from 37 to 18 for California and from 10 to 6 for Italy) with respect to those detected by Lolli and Gasperini (2006).

3. Analysis

We estimated the parameters of each decay model by the maximum likelihood method (Ogata, 1988). To maximize the likelihood we use an algorithm (Lolli et al., 2009) that combines a random search over a reasonable interval of the parameters space and Newton-like optimizations (Dennis and Schnabel, 1983) of the best random solutions. We estimate the parameters of our sequences both with and without the inclusion of the constant background term μ and by considering different lengths of the observing interval of 3, 6, 12, 24 and 48 months.

We compare the goodness-of-fit of the different decay models by three criteria: the corrected Akaike Information Criteria (AIC_c, Akaike, 1974; Hurvich and Tsai, 1989), the corrected Bayesian Information Criteria (BIC, Schwarz, 1978; Draper, 1995) and the simple maximum log-likelihood function $l_{\max}$. For the AIC_c and BIC we adopt (consistently with Lolli and Gasperini, 2006) the following scores

$$AIC_c = l_{\max} - k - \frac{k(k+1)}{n-k-1} \quad (5)$$

$$BIC = l_{\max} - \frac{k}{2}\ln\frac{n}{2\pi} \quad (6)$$

where k is the number of free parameters (3 for the MOM, 4 for MSE and LPL and one more for all models when the background rate μ is considered), and n is the number of data (the number of aftershocks in each sequence). With these formulations, which differ from the usual ones for the sign and for a factor of 2, the best model is the one giving the highest score.

In the following comparisons, we will also consider $l_{\max}$ because we might hypothesize that the additional parameter of the MSE and LPL, which models the exponential decay, might not be able improve significantly the fit (and increase correspondingly the log-likelihood function) when the length of the observing time interval (the assumed duration of the sequence) is short with respect to the relaxation time (t_0 for the MSE and about t_a for the LPL). In these