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Convection in a spherical shell heated by an isothermal core and internal sources: Implications for the thermal state of planetary mantles

M. Hosein Shahnas[a,1], Julian P. Lowman[a,*], Gary T. Jarvis[b], Hans-Peter Bunge[c]

[a] Department of Physical and Environmental Sciences, University of Toronto Scarborough, Toronto M1C 1A4, Canada
[b] Department of Earth and Space Science and Engineering, York University, Toronto M3J 1P3, Canada
[c] Department of Earth and Environmental Sciences, Ludwig-Maximilians University Munich, Theresienstrasse 41, Munich DEU D-8033, Germany

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ABSTRACT

The parallelized three-dimensional spherical convection code, TERRA, is employed to study the mean temperatures and planforms of convecting planetary mantles in spherical shell geometries. We vary the factor f which controls the degree of curvature, defined as the ratio of the radii of the inner and outer bounding surfaces, the Bénard-Rayleigh number, Ra_B, and the dimensionless rate of internal heating, H. We develop parameterized expressions for predicting the mean temperature of convecting spherical shells which are heated partially from within and partially from below by a hot isothermal lower boundary. Our parameterization is fit to a data set of mean temperatures from 23 numerical model calculations for $f = 0.547$ (appropriate to Earth's mantle). We then demonstrate that this parameterization of mean temperature in terms of f, Ra_B and H extends to other values of f as well. For all values of f, Ra_B and H considered in this study, our predicted mean temperatures agree with the model calculations to within 2.4%. The scaling analysis is extended to obtain an expression for surface heat flux in terms of Ra_B and H for $f = 0.547$. In that case we obtain a predictive equation for surface heat flux that agrees to within 11% of the observed values. Our findings provide a useful tool for parameterizing the temperature and surface heat flux of planetary mantles of varying geometry and heating configurations.

1. Introduction

Some knowledge of the composition, thickness and heating rate of a terrestrial planet's mantle is required for even the most rudimentary model of its thermal structure. The possibilities for these fundamental parameters allows for a diverse range of thermal states, even in a simple medium featuring uniform properties and boundary conditions (Bercovici et al., 1989; Schubert et al., 1993). Previous studies of uniform property fluids have shown that despite the apparent complexity of the obtainable convective planforms, simple predictive equations can be derived for bulk characteristics such as the mean temperature; both in plane layer systems, featuring heating from within and below (Sotin and Labrosse, 1999), and axi-symmetric spherical shell systems, featuring heating by an isothermal core (Vangelov and Jarvis, 1994). The findings of these studies have been extended to derive theoretical but untested predictive equations for the mean temperature of an infinite Prandtl number, uniform property, convecting fluid in a random thickness spherical shell heated by both internal sources and an isothermal core (Sotin and Labrosse, 1999). The goal of the study presented here is to test and refine these equations to provide a validated parameterization for predicting mean temperatures.

Studies (Jarvis et al., 1995) of solely bottom heated fluid spherical shells have shown that the number of convection cells and the time dependence of the flow is dependent on the ratio, f, of the inner and outer shell radii. Convection planform and time dependence are also dependent on the rate of internal heating, H, and the Bénard-Rayleigh number, Ra_B, of the system (a measure of the vigour of the flow driven by bottom heating). Because convection cell planform also affects mean temperature (Jarvis et al., 1995) it is not obvious that a simple relation between the parameters, H and Ra_B, and the global temperature should be obtainable.

We obtain numerical solutions for the temperature fields in isoviscous spherical shell models of planetary mantles heated by an isothermal boundary condition at the core and by uniformly distributed internal sources. The models are cooled from above by an isothermal surface. The specification of an isothermal condition at the core assumes that the planet has a liquid core (or outer core) and is therefore able to mix on much more rapid timescales than

* Corresponding author at: Department of Physics, University of Toronto, Toronto, ON M5S 1A7, Canada. Tel.: +1 416 208 4880; fax: +1 416 287 7279.
E-mail address: lowman@utsc.utoronto.ca (J.P. Lowman).
[1] Also at: Department of Physics, University of Toronto, Toronto, ON M5S 1A7, Canada.

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the mantle. We consider a range of model geometries (determined by the ratio of the inner radius to the outer radius of the spherical shell, f, the curvature factor) but initially focus on models that feature the Earth's curvature factor, $f = 0.547$. We compare our results in this fixed geometry to previous findings in a Cartesian geometry ($f = 1$) in order to refine previously proposed equations for a generalized curvature factor. In addition to working towards a single equation for predicting the global thermal characteristics of a planetary mantle with arbitrary f and heating mode, we examine the effect of these parameters in causing transitions in the convective planform of the systems.

2. Modelling method

We model infinite Prandtl number convection in a spherical shell with radially inward directed gravity using the parallelized code TERRA (Bunge and Baumgardner, 1995; Bunge et al., 1996, 1997; Phillips and Bunge, 2005). In order to compare our findings with the most relevant previous studies (e.g., Sotin and Labrosse, 1999) we employ the Boussinesq approximation and suppress the effects of compressibility. We focus on isoviscous convection heated by both core heat loss and internal mantle heat sources. The rate of internal heating is constant in time. Isothermal boundary conditions are specified at the inner shell radius, R_i, and outer shell radius, R_o. The temperature difference across the shell thickness, $d = R_o - R_i$, is ΔT. The vigour of convection driven by the temperature difference of the bounding surfaces of the spherical shell can be measured by the Bénard-Rayleigh number:

$$Ra_B = \frac{g\alpha\Delta T d^3}{\kappa\nu}, \tag{1}$$

where g is the gravitational acceleration, α is the thermal expansivity, κ is the thermal diffusivity and ν is the kinematic viscosity.

Typical values of the parameters defining Ra_B indicate that the mantles of Earth and other terrestrial planets are characterised by Bénard-Rayleigh numbers that are well above the critical value (McKenzie et al., 1974; Schubert et al., 2001). However, the source of most of the heat flux from Earth's mantle is the concentration of radiogenic elements in the mantle (Schubert et al., 1980). Significant heating in other planets with a similar make-up to the Earth likely also comes from internal mantle heat sources. The vigour of convection driven by internal sources can be specified in terms of an internal heating Rayleigh number:

$$Ra_H = \frac{g\alpha\phi d^5}{\kappa k\nu}, \tag{2}$$

where k is the thermal conductivity and ϕ is the rate of internal heat generation per unit volume (Roberts, 1967).

We non-dimensionalise the system of equations governing convection in the spherical shell in terms of the diffusion time across the shell thickness, so that dimensional times are recovered from non-dimensional times by multiplying by d^2/κ. In its non-dimensional form the equation for the conservation of heat in the system described is thus

$$\frac{\partial T}{\partial t} = \nabla^2 T - \mathbf{v}\cdot\nabla T + \frac{Ra_H}{Ra_B}, \tag{3}$$

where T is the non-dimensional temperature, t is the non-dimensional time and $\mathbf{v}$ is the non-dimensional velocity. Ra_H/Ra_B is equivalent to the non-dimensional internal heating rate, H.

The equations describing the conservation of mass and momentum as well as the standard linearised equation of state complete the system of equations describing flow evolution in the fluid modelled. Adopting the non-dimensionalisation of time described above, these equations take the form

$$\nabla\cdot\mathbf{v} = 0, \tag{4}$$

$$\nabla^2\mathbf{v} - \nabla P = Ra_B T\hat{\mathbf{z}}, \tag{5}$$

and

$$\rho = \rho_0[1 - \alpha T], \tag{6}$$

respectively, where $\hat{\mathbf{z}}$ is a unit vector parallel to the direction of gravitational acceleration, P is the non-dimensional pressure and ρ_0 is the density at the surface of the spherical shell which has a non-dimensional temperature of zero. Ra_B and H are thus the sole fluid parameters governing the solution of the system. (Note that f is an independent geometric parameter which also governs the evolution of the system.)

We solve the continuity and momentum equations specifying free-slip surfaces at R_i and R_o. The radial and lateral resolution of the numerical mesh employed in each of the calculations performed are adjusted according to the value of the specified Bénard-Rayleigh number.

The combinations of Rayleigh number, Ra_B, internal heating rate, H, and curvature factor, f, for all of the cases examined in this study, result in time-dependent convection. The mean temperatures and heat fluxes that we quote in the following sections are therefore temporal averages determined once each model has reached a statistically steady state. We consider the solution to have reached such a condition once the mean temperature of the system is no longer showing any clear long-term heating or cooling trends. The average values that we quote, along with the standard deviation of the time series, are obtained over multiple mantle overturn times once a steady condition has been attained. The initial conditions for our solutions are specified as mildly perturbed fields featuring steep temperature gradients at the solution domain boundaries and isothermal regions between the boundary regions. The temperature of the isothermal bulk of the fluid is adjusted at the start of each model run in accord with the heating rate of the calculation. We specify higher initial mean temperatures for calculations with relatively high internal heating rates. As a result, collectively, the calculations heated or cooled to their statistically steady state much more quickly than they would have if we have used the same initial condition for all cases.

3. Results

We initially present results from 23 numerical models with a fixed ratio, f, of $R_i/R_o = 0.547$ (the ratio of the Earth's outer core radius to its mean surface radius). For this geometry, we examine the thermal characteristics in calculations featuring Bénard-Rayleigh numbers ranging from 10^4 to 10^7 and H between 2.353 and 47.064. (The upper value of this range for H is approximately twice estimates of the effective current rate of heating in the Earth's mantle that results from internal sources and secular cooling (Schubert et al., 2001). In combination, these heat sources, which mimic each other (Krishnamurti, 1968; Weinstein and Olson, 1990), may supply about 0.8 of the Earth's mean surface heat flux.) The non-dimensional heating rate, H, is dependent on d^2; consequently, modelling planets with shallower mantles than the Earth requires specifying lower values for H as well as Ra_B. We model planets with low values of Ra_B and H for an Earth-like value of f in order to obtain thorough coverage of $Ra - H$ parameter space. Table 1 summarizes the heating mode specification, grid resolution and non-dimensional mean temperature, θ, of each model solution. (Other columns of data appearing in the table are discussed in later sections). Table 1 also introduces a naming convention for our models that is based on the Rayleigh number and a multiplicative factor