Physics of the Earth and Planetary Interiors 175 (2009) 53–62

Contents lists available at ScienceDirect

Physics of the Earth and Planetary Interiors

journal homepage: www.elsevier.com/locate/pepi

Observations of ultra-long period normal modes from the 2004 Sumatra–Andaman earthquake

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ARTICLE INFO

Article history:
Received 14 August 2007
Accepted 6 February 2008

Keywords:
Sumatra-Andaman earthquake
Normal modes
Splitting

ABSTRACT

The great December 2004 Sumatra–Andaman earthquake was the first "giant" or "extreme" (moment magnitude $M_w \geq 9$) earthquake recorded by broadband digital seismometers whose data were rapidly available to investigators worldwide. As a result, analysis of the earth's longest period normal modes became a primary tool for studying the earthquake, rather than an elegant afterthought. The mode data provided the first evidence that the earthquake was much larger ($M_w \approx 9.3$) than initially inferred from surface wave data and involved slip on a much longer fault than initially inferred from body wave data. These observations in turn yielded important insight into the likely recurrence of similar earthquakes and the resulting tsunamis both on the segment of the trench that ruptured and on neighboring segments. The normal mode data are more numerous and much higher quality than previously available. They thus provide the first direct evidence for effects that had been theoretically predicted, such as the control of the splitting pattern by receiver latitude and the splitting of torsional modes. They similarly yield better results for mode properties such as the attenuation of the longest period radial modes, found in agreement with existing models of intrinsic Earth attenuation.

1. Introduction

Seismological advances often involve three factors. First, a suitable earthquake must generate a signal of interest. Second, the resulting signal must be recorded by appropriate seismometers. Third, the theoretical framework needed to interpret the signal needs to be available.

Studies of the earth's normal modes followed this pattern (Fig. 1). Benioff (1958) and Båth (1958) had hinted that ultra-long period oscillations observed in the time domain following the 1952 Kamchatka and the (much smaller) 1958 Fairweather, Alaska earthquakes might have represented free oscillations of the Earth. However, the undisputed observation and cataloguing of the planet's normal modes, including the gravest, ${}_0S_2$ ($T = 3232$ s), had to wait until the great 1960 Chilean earthquake (Alsop et al., 1961; Benioff et al., 1961; Ness et al., 1961). By then, improved strainmeters and gravimeters had been developed that could record ground motions with periods much longer than possible with existing seismometers, while progress in analytical techniques made it feasible to obtain reliable measurements directly in the frequency domain. In this respect, the 1960 Chilean event had perfect timing. By the time of the great "Good Friday" Alaskan earthquake of 28 March 1964, the newly deployed WWSSN seismometers provided normal mode data at periods of several hundred seconds (Dziewonski and Gilbert, 1972), although they were not specifically designed for this application.

By contrast, no truly great earthquake ($M_0 > 10^{29}$ dyn cm) occurred for nearly 40 years after the 1965 Rat Island event (Fig. 1), a time window corresponding grossly to one human generation. This period witnessed several technological revolutions pertinent to seismological research: the acceptance of Plate Tectonics, the explosion of information technology, and the development and generalization of networks of long-period digital instruments, such as the IDA gravimeter network (Agnew et al., 1976) and later the broad-band GEOSCOPE and IRIS Global Seismographic Networks. In this respect, the 2004 Sumatra–Andaman earthquake finally offers a long-overdue combination of the three necessary ingredients for advances in seismological research, especially in the field of the Earth's free oscillations.

2. Theoretical background

With the development of analytical theory and tools (e.g., Alterman et al., 1959), the observation of the Earth's modes in 1960 opened the way for their use in long period studies of earthquake sources and Earth structure. These two effects are naturally separated in the normal mode formulation. In Saito's (1967) notation using spherical coordinates with the earthquake at the pole, the

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doi:10.1016/j.pepi.2009.03.002

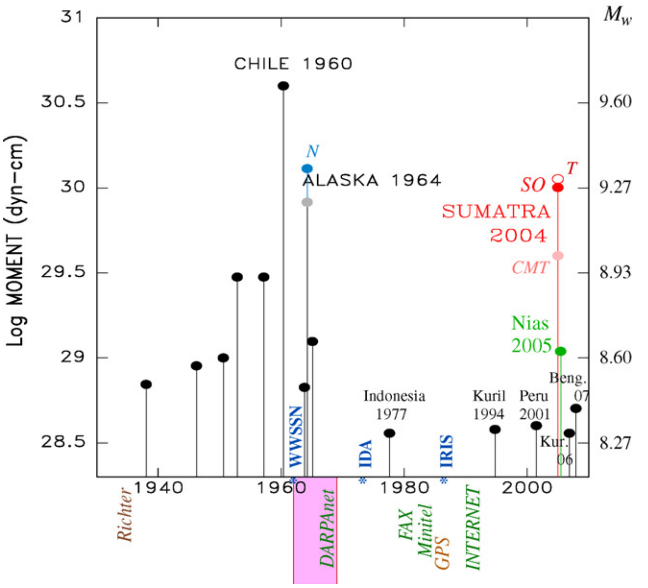

Fig. 1. Earthquakes with moments $M_0 \geq 3.5 \times 10^{28}$ dyn cm recorded in the past 70 years. Note the absence of very large events between 1964 (Alaska) and 2004 (Sumatra). Moment estimates of the latter refer to the CMT catalog, and to values published by Stein and Okal (2005) (solid dot; *SO*) and Tsai et al. (2005) (open circle; *T*). *N* shows the reassessment of the 1964 Alaska source by Nettles et al. (2005). The stars on the horizontal axis illustrate progress in seismic instrumentation. Vertical labels similarly identify Richter's (1935) introduction of the concept of magnitude, as well as milestones in information technology. The shaded band corresponds to the advent of plate tectonics.

three-dimensional seismic displacement field $\mathbf{u}(r, \theta, \phi)$ is expanded as a sum of normal modes described by radial order n, angular order l, and azimuthal order m. For spheroidal normal modes, which involve radial (vertical) and transverse motions of the Earth analogous to *P*–*SV* or Rayleigh waves,

$$\boldsymbol{u}^S(r, \theta, \phi) = (u_r, u_\theta, u_\phi)$$

$$= \sum_n \sum_l \sum_{m=-l}^{l} {}_nA_l^m [{}_nU_l(r)\boldsymbol{R}_l^m(\theta, \phi) + {}_nV_l(r)\boldsymbol{S}_l^m(\theta, \phi)] e^{i_n\omega_l^m t} e^{-({}_n\omega_l^m t/2{}_nQ_l^m)}, \quad (1)$$

involving combinations of the spherical harmonics $Y_l^m(\theta, \phi)$

$$\boldsymbol{R}_l^m = (Y_l^m, 0, 0) \quad (2)$$

$$\boldsymbol{S}_l^m = \left(0, \frac{\partial Y_l^m(\theta, \phi)}{\partial \theta}, \frac{1}{\sin \theta} \frac{\partial Y_l^m(\theta, \phi)}{\partial \phi}\right)$$

weighted by radial eigenfunctions ${}_nU_l(r)$ for vertical motion and ${}_nV_l(r)$ for horizontal motion.

Similarly, for torsional normal modes, which involve horizontal motions of the Earth analogous to *SH* or Love waves, the displacement vector is written

$$\boldsymbol{u}^T(r, \theta, \phi) = \sum_n \sum_l \sum_{m=-l}^{l} {}_nB_l^m {}_nW_l(r) \; \boldsymbol{T}_l^m(\theta, \phi) e^{i_n\omega_l^m t} e^{-({}_n\omega_l^m t/2{}_nQ_l^m)}. \quad (3)$$

where

$$\boldsymbol{T}_l^m = \left(0, \frac{1}{\sin \theta} \frac{\partial Y_l^m(\theta, \phi)}{\partial \phi}, \frac{-\partial Y_l^m(\theta, \phi)}{\partial \theta}\right) \quad (4)$$

The modes' radial eigenfunctions and eigenfrequencies ${}_n\omega_l^m$ are determined by the earth's velocity and density structure (Alterman et al., 1959). Similarly, the modes' attenuation is expressed through their quality factor ${}_nQ_l^m$, that depends on the distribution of anelasticity in the Earth (Sailor and Dziewonski, 1978). (The values of ${}_n\omega_l^m$ and ${}_nQ_l^m$ are different for spheroidal and torsional modes sharing the same indices, but to simplify the notation, we use the same symbols, as the nature of the mode involved is usually evident from the context.) Following an earthquake, the displacement contributed by each mode is determined by its excitation amplitudes ${}_nA_l^m$ or ${}_nB_l^m$, that depend on the depth, geometry, and time history of the seismic source (Saito, 1967; Gilbert, 1970).

In this context, normal mode data are used for studies of both Earth structure, by focusing on ω and Q (e.g., Gilbert and Dziewonski, 1975), and earthquake sources by focusing on the parameters A and B (e.g., Kanamori and Cipar, 1974). Such studies are typically termed "normal mode" studies when they consider individual modes, and "surface wave" studies when they treat a set of modes as a continuous spectrum, usually through the use of an asymptotic expansion of the spherical harmonics to express travelling waves (Kanamori and Stewart, 1976).

While normal mode studies continued during the age of mega-earthquake quiescence (1965–2004), they had to be confined to smaller, if still "great" ($M_W \geq 8$), earthquakes such as the 1977 Indonesia, 2001 Peru, or deep 1994 Bolivia events. Most studies (e.g., Buland et al., 1979; Geller and Stein, 1979; Riedesel et al., 1980; Stein and Nunn, 1981; Widmer et al., 1992) focused on measuring eigenfrequencies and attenuation, although some addressed source properties (e.g., Ekström, 1995; Okal, 1996). The latter typically were conducted well after the earthquake and focused on refining a source model developed from body and surface wave data.

3. Previous results on the Sumatra earthquake

The rapid availability of normal mode data following the December 26, 2004 Sumatra–Andaman (or "Sumatra") earthquake led to a new approach. This event was the first $M_W \geq 9$ earthquake since the 1964 Alaskan event. Its enormous size and its devastating tsunami promoted a wide range of studies by Earth scientists around the world, greatly facilitated by the availability in near-real time of high quality seismological, geodetic, and other geophysical data. Information became rapidly available, making this the best studied earthquake of its size, and providing a basis for studies that will likely continue for many years.

Focusing initially on ultra-long period ($T > 500$ s) observations of normal modes from the Global Seismic Network, we showed in Stein and Okal (2005) that the earthquake was much larger and involved slip on a much longer fault than at first thought. This analysis, published 3 months after the earthquake, provided insight into the generation of the tsunami, the recurrence time of similar earthquakes, and the regional tectonics. A key result was that, because the entire aftershock zone slipped, strain accumulated from subduction of India beneath the Burma microplate (or sliver) along the northern part of the rupture had also been released. This left no immediate danger of a similar oceanwide tsunami being generated by slip on that segment of the plate boundary. Because of the complexity of the local tectonic regime (e.g., the northern extent of the Burma sliver is not precisely known), the eventuality of a megathrust earthquake immediately to the north of the 2004 rupture cannot be totally discounted (Okal and Synolakis, 2008). Conversely, we pointed out in Stein and Okal (2005) the possibility of a large earthquake on the neighboring trench segment to the south, a scenario described at the same time in greater detail by McCloskey et al. (2005), in the framework of Coulomb stress transfer. Two such events took place, the Simeulue-Nias earthquake ($M_W = 8.7$) on March 28, 2005, and the Bengkulu earthquake