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Multi-item EOQ model with hybrid cost parameters under fuzzy/fuzzy-stochastic resource constraints: A geometric programming approach

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ABSTRACT

In this paper, a multi-item economic order quantity (EOQ) model is considered in which the cost parameters are of fuzzy/hybrid nature under two types of resources — (a) resources as fuzzy quantities; (b) resources as fuzzy and fuzzy-random quantities. The unit cost depends on demand rate. The time horizon is taken to be infinite. We find the average cost for the model, which is a function of order quantity and demand rate and also of some hybrid parameters. When the resources are fuzzy quantities, the problem is transformed into its equivalent unconstrained deterministic form by using a surprise function for the constraints. The problem involving hybrid number is again equivalently rewritten as a multi-objective (minimization of the mean of the objective function and variance function of the distribution) inventory problem. Introducing new variables we transform the terms of the functions into signomial types. Using fuzzy multi-objective solution procedure we solve the problem through Geometric Programming approach. Sensitivity analysis has been performed to study the effect of different weights considered for mean objective function and variance function.

1. Introduction

In most of the economic order quantity (EOQ) models, researchers considered unit purchase cost as constant. This assumption is not true in reality. Unit cost is dependent on demand. In an EOQ system the number of units ordered to meet the demand is demand dependent. So, to meet high demand, order for more quantities is placed. When the ordered quantity is high, generally some discount is offered to purchase the items. Therefore the unit cost will be reduced if the number of units ordered is large and vice versa. Cheng [1,2] developed some inventory models in crisp environment with demand dependent unit cost and solved them using geometric programming (GP) technique. This investigation did not include the fuzzy-stochastic environment and constraints on the system, which have been taken into account in the present analysis.

In reality, the available information can be deterministic, probabilistic or fuzzy. This information is concerned with the mathematical models in the area of management sciences. Some parameters in inventory problems normally are not crisp ones but their values occur according to some probability distribution and are modeled as random variables. Some other parameters, in the industrial scenario, may be understood and/or described by the decision maker with vagueness and this uncertainty is different from the uncertainty in stochastic sense. In order to incorporate this type of uncertainty

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(non-stochastic sense), fuzzy set theories are used. To this end, the pioneers of stochastic programming were Kall [3], Vajda [4], etc. and fuzzy linear programming was extensively developed by Bellman et al. [5], Dubois et al. [6], Zadeh [7], Zimmermann [8] etc.

But in many practical situations, knowledge about the data (such as cost parameters in inventory problems) is neither purely probabilistic nor possibilistic but rather a blend of both kinds (cf. [9]). Suppose for example, 'there is a 80% chance in high quality production from a particular machine'. Here, the concept of '80% chance' is borrowed from stochastic sense whereas the linguistic 'high quality' comes from possibilistic sense. A blend of several types of information is encountered in realistic models to furnish an excellent depiction of the phenomenon which leads to the concept of hybridization. This means that randomness and impreciseness can be combined simultaneously to represent the real world as it is perceived. Such combinations may be represented by hybrid numbers, random fuzzy numbers, random fuzzy subsets, expectation of fuzzy sets, possibility of random variables, and several others. These novel concepts will be described in view of applications in human sciences but such tools can be also used in every scientific research such as operations research, etc. Again a parameter may have different fuzzy values in nature with some non-fuzzy probabilities. These parameters are called fuzzy-random parameters. For example, a company may have different securities, share etc. and by selling these they may raise their capital for budget investment to buy new products which are to be supplied. Since the share market is probabilistic, the amount of money extracted from the market is random. The amount may be "around $10 million with probability 0.3", "about $15 million with probability 0.5" etc.

Also EOQ models deal with the minimization of the sum of set-up cost, production/procurement cost, holding cost etc. Recently, Mandal et al. [10] investigated a multi-objective fuzzy inventory model with three constraints using fuzzy geometric programming approach. They formulated the model with fuzzy parameters only in fuzzy environment and then solved it using modified geometric programming technique. But, an organization must face the changing scenario where the values of different cost parameters are changing in different ways. In general the parameters may be uncertain in nature in fuzzy-stochastic sense. This variation may happen in two different ways; (i) some parameters may vary in such a way that a part is fuzzy and another part is random in nature.These parameters are called hybrid parameters. Here we consider the cost parameters as hybrid numbers; (ii) some parameters may be fuzzy random i.e. they become fuzzy with some probability. We consider here some resources as fuzzy-random quantities.

We consider here two types of models. For the first model, cost parameters are hybrid and resources are fuzzy; in the second, the cost parameters are hybrid and resources are fuzzy and fuzzy random.

Some fuzzy constraints can be transformed into its equivalent deterministic forms by introducing a surprise function (cf. [11]). Lodwick and Bachman [12] used surprise function technique to convert a fuzzy possibility/necessity optimization problem into a deterministic one. By this technique, the objective function becomes of rational form. Then introducing new variables, the terms of the problem are converted to signomial types and solved by generalized geometric programming method (cf. [13]). Geometric programming technique has some distinctive features with respect to nonlinear optimization of posynomial/signomial functions. This method has been applied to different fields of Science and Engineering. Several authors have used this in inventory control system also. Cao [14,15] developed fuzzy posynomial geometric programming technique along with its duality for the solution of single and multi-objective problems using L-R fuzzy coefficients. Liu [16, 17] also solved some fuzzy machining models with fuzzy exponents and coefficients based on the duality of geometric programming and by using a variable substitution technique. Roy and Maiti [18] used geometric programming technique to solve both single item and multi-item fuzzy inventory problems.

But, the models mentioned above deal with the fuzzy coefficients/parameters only. None has considered the blended models taking both fuzzy and fuzzy-random (hybrid) coefficients/parameters into account. Moreover, in some formulations, constraints, if any, have been dealt with conventionally. In this present model, both fuzzy and hybrid numbers are taken as coefficients/parameters and the constrained problem is converted to an unconstrained one using a surprise function in possibility sense.

The objective of this paper is to provide a framework to an EOQ model in fuzzy-stochastic environment. Here, cost per unit of an item is dependent on demand. The cost parameters are not precise in general. They are represented by numbers which are both fuzzy and random with a certain probability density function, known as hybrid numbers. In space and budget constraints, total available space of the warehouse and the total budget are considered here as fuzzy and fuzzy/fuzzy random respectively. Finally each model is converted into a multi-objective inventory problem, which minimizes the mean total cost and also minimizes the total dispersion value of the cost function. The total cost and its dispersion may fluctuate within some range i.e. the goals of the objective functions are fuzzy in nature. Following the procedures of crispitization of fuzzy and fuzzy-random parameters and reducing the multi-objective optimization problem to a single objective optimization one using a surprise function, the problem is solved by GP method. Numerical examples are given to illustrate the models. The sensitivity analysis is presented due to change of the preference values of the objective functions.

2. Generalized geometric programming method

The general constrained signomial geometric programming problem is of the following form:

Primal problem:

$$\text{Min } f_0(x) \tag{1}$$