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An integrated aircraft routing, crew scheduling and flight retiming model

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Abstract

In the integrated aircraft routing, crew scheduling and flight retiming problem, a minimum-cost set of aircraft routes and crew pairings must be constructed while choosing a departure time for each flight leg within a given time window. Linking constraints ensure that the same schedule is chosen for both the aircraft routes and the crew pairings, and impose minimum connection times for crews that depend on aircraft connections and departure times. We propose a compact formulation of the problem and a Benders decomposition method with a dynamic constraint generation procedure to solve it. Computational experiments performed on test instances provided by two major airlines show that allowing some flexibility on the departure times within an integrated model yields significant cost savings while ensuring the feasibility of the resulting aircraft routes and crew pairings.

Keywords: Aircraft routing; Crew scheduling; Flight retiming; Integrated planning; Time windows; Benders decomposition; Column generation

0. Introduction

Airlines usually use a sequential procedure to plan their operations (see, e.g. [1]). By solving a *flight scheduling problem*, they first create a schedule that specifies each flight leg to be flown during a given period and sets departure and arrival times for each of those legs. Then, the *fleet assignment* is performed to assign an aircraft type to each flight leg to maximize anticipated profits while taking into account the number of available aircraft. For each aircraft type, an *aircraft routing problem* is then solved to determine the sequence of flight legs to be flown by each individual aircraft so that each leg is covered exactly once while ensuring appropriate aircraft maintenance. With the aircraft routes on hand, the airline then builds crew rotations or *pairings* by solving a *crew scheduling problem* for each aircraft type. A pairing is a sequence of duty periods separated by overnight rests, and a *duty period* is a sequence of flight legs separated by smaller rest periods, called *sits* (or *connections*). The objective of the crew scheduling problem is to determine a minimum-cost set of pairings so that every flight leg is assigned a qualified crew and every pairing satisfies the set of applicable work rules. For example, each duty period in a pairing must respect limits on total work time, total flight time and total number of landings. In the last step of the planning process, pairings are finally combined to form personalized monthly schedules that are assigned to employees by solving a *crew bidding problem* or a *crew rostering problem*.

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While a sequential procedure greatly simplifies the process, Cordeau et al. [2], Klabjan et al. [3] and Cohn and Barnhart [4] have shown that integrating the aircraft routing and crew scheduling problems can generate solutions that are significantly better than those obtained by solving the problems sequentially. Because the minimum connection time required between two successive flight legs covered by the same crew depends on whether the same aircraft is used on both legs, aircraft routing decisions have an impact on the set of feasible pairings. Consequently, a sequential planning procedure is likely to yield suboptimal solutions. A connection that is too short to be made by a crew when the two associated legs are not flown by the same aircraft is said ot be *short*. In this paper, we consider an additional level of integration by adding some flight scheduling decisions to the integrated aircraft routing and crew scheduling problem. More precisely, the departure time of each flight leg is allowed to deviate slightly from the planned schedule. Obviously, the same departure time has to be chosen for both the aircraft and the crews, and this complicates the problem. However, an integrated approach can take advantage of the added schedule flexibility to a greater extent since the departure times are chosen by taking into account the benefits to both the aircraft routings and the crew pairings. This would not be possible with a sequential solution process in which modifying the schedule in one step could have unforeseen consequences on the next step. When only small modifications from the original flight schedule are considered, it is reasonable to assume that flight demand does not change significantly (see, e.g. [3,5,6]).

Several modeling and solution approaches have been proposed to separately address the aircraft routing and crew pairing problems. The former problem was studied, among others, by Daskin and Panayotopoulos [7], Feo and Bard [8], Clarke et al. [9], Gopalan and Talluri [10] and Talluri [11]. Numerous contributions regarding the different variants of the crew scheduling problem can also be found in the operations research literature. For an overview, the reader is referred to the recent survey of Barnhart et al. [12]. Issues related to the introduction of maintenance and crew considerations in the fleet assignment problem were discussed by Clarke et al. [13], Rushmeier and Kontogiorgis [14] and Barnhart et al. [15]. Finally, other interesting contributions with respect to the integration of the planning process are the approaches presented by Desaulniers et al. [5] and Barnhart et al. [16] for the combined fleet assignment and aircraft routing problem.

In recent years, there has been a growing interest in the integration of aircraft routing and crew scheduling problems. Cordeau et al. [2] have introduced a model that integrates the complete aircraft routing and crew pairing formulations to which is added one linking constraint for each short connection. To handle these linking constraints, a solution approach based on Benders decomposition is used. The solution process iterates between a master problem that solves the aircraft routing problem, and a subproblem that solves the crew pairing problem. Short connections are fixed by the master problem and the subproblem constructs minimum-cost crew pairings using only the fixed set of short connections. Because of their particular structure, both of these problems are solved by column generation. On a set of test instances based on data provided by a Canadian airline, the integrated approach reduced variable crew costs by 9.4% with respect to the sequential planning process commonly used in practice.

The latter model was further enhanced by Mercier et al. [17] who have introduced a generalized formulation in which solution robustness is improved by penalizing connections that are likely to introduce delays if they are not performed by the same aircraft. The authors also show that reversing the order of the solution sequence, i.e., solving the crew pairing problem in the Benders master problem as opposed to the aircraft routing problem, yields important improvements over the approach of Cordeau et al. [2]. Most costs in the integrated problem are associated with the crew pairings and, by reversing the natural solution sequence, the aircraft routing subproblem is mostly transferring feasibility information to the master problem and very little optimality (or cost) information. This results in a significant decrease in the number of Benders cuts generated. The identification of Pareto-optimal cuts was also shown to be useful in further improving the speed of convergence.

Cohn and Barnhart [4] have also proposed an integrated model, but instead of incorporating the aircraft routing formulation in the model, variables representing complete solutions to the aircraft routing problem are used in an *extended crew pairing model*. This obviously reduces the number of constraints, but may lead to a very large number of additional variables. The authors show that only a subset of the feasible aircraft routing solutions needs to be included in the model, i.e., one column for each *unique and maximal maintenance-feasible short connection set*. These columns can be generated individually and sequentially, in a preprocessing step, by solving a series of aircraft routing problems with additional constraints and a modified objective function. They also propose to solve the extended crew pairing problem by a branch-and-price algorithm in which both crew pairing and aircraft routing solutions are generated dynamically.

Klabjan et al. [3] have presented a partially integrated approach that solves the crew pairing problem first, but includes additional constraints that count the number of available aircraft on the ground at any time. Under the assumption that