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The effects of buoyancy on shear-induced melt bands in a compacting porous medium

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ABSTRACT

It has recently been shown [Holtzman, B., Groebner, N., Zimmerman, M., Ginsberg, S., Kohlstedt, D., 2003. Stress-driven melt segregation in partially molten rocks. Geochem. Geophys. Geosyst. 4, Art. No. 8607; Holtzman, B.K., Kohlstedt, D.L., 2007. Stress-driven melt segregation and strain partitioning in partially molten rocks: effects of stress and strain. J. Petrol. 48, 2379–2406] that when partially molten rock is subjected to simple shear, bands of high and low porosity are formed at a particular angle to the direction of instantaneous maximum extension. These have been modeled numerically and it has been speculated that high porosity bands may form an interconnected network with a bulk, effective permeability that is enhanced in a direction parallel to the bands. As a result, the bands may act to focus mantle melt towards the axis of mid-ocean ridges [Katz, R.F., Spiegelman, M., Holtzman, B., 2006. The dynamics of melt and shear localization in partially molten aggregates. Nature 442, 676–679]. In this contribution, we examine the combined effects of buoyancy and matrix shear on a deforming porous layer. The linear theory of Spiegelman [Spiegelman, M., 1993. Flow in deformable porous media. Part 1. Simple analysis. J. Fluid Mech. 247, 17–38; Spiegelman, M., 2003. Linear analysis of melt band formation by simple shear. Geochem. Geophys. Geosyst. 4, doi:10.1029/2002GC000499, Article 8615] and Katz et al. [Katz, R.F., Spiegelman, M., Holtzman, B., 2006. The dynamics of melt and shear localization in partially molten aggregates. Nature 442, 676–679] is generalized to include both the effects of buoyancy and matrix shear on a deformable porous layer with strain-rate dependent rheology. The predictions of linear theory are compared with the early time evolution of our 2D numerical model and they are found to be in excellent agreement. For conditions similar to the upper mantle, buoyancy forces can be similar to or much greater than matrix shear-induced forces. The results of the numerical model indicate that bands form when buoyancy forces are large and that these can significantly alter the direction of the flow of liquid away from vertical. The bands form at angles similar to the angle of maximum instantaneous growth rate. Consequently, for strongly strain-rate dependent rheology, there may be two sets of bands formed that are symmetric about the direction of maximum compressive stress in the background mantle flow. This second set of bands would reduce the efficiency with which melt bands would focus melts towards the ridge axis.

1. Introduction

Holtzman et al. (2003) and Holtzman and Kohlstedt (2007) presented experiments in which partially molten ductile rocks were subjected to deformation approximating simple shear. It was found that if the size of the sample was similar to or greater than the compaction length, high porosity bands would spontaneously form at angles of roughly 20° to the shear plane. If a sample of a partially molten material is larger than its compaction length, a significant degree of matrix deformation will take place if the sample is subjected to an applied stress (McKenzie, 1984). Stevenson (1989) had shown theoretically that partially molten materials should spontaneously segregate into high and low porosity regions provided that the viscosity of the solid matrix is a decreasing function of porosity when the matrix is subjected to pure shear. The formation of bands, perpendicular to the direction of maximum extension, in simulations of shear flow in strain independent but porosity weakening rheology, was demonstrated by Richardson (1998). Richardson (1998) also included the effects of buoyancy and showed that veins formed when buoyancy was active and that background shear resulted in the elongation in the direction of maximum compressive stress of a rising porosity solitary wave. Spiegelman (2003) and Katz et al. (2006) showed using linear theory and numerical simulations that such bands will grow and that

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the angle at which the bands grow fastest is a function of the strain-rate dependence of the viscosity of the matrix. They found that if the exponent relating the viscosity to the strain rate is roughly four or greater, that the melt bands form at low angles similar to the ones seen in the experiments.

Melting is believed to occur in roughly the top 60 km in the upwelling region below the ridge axis (Hess, 1992). The lateral extent of melting is believed to be roughly 100 km (Forsyth et al., 1998) while melts are mostly extracted within 1 km of the ridge axis (Vera et al., 1990). A number of mechanisms have been proposed to explain the focusing of mantle melt towards the ridge axis. These include the stresses imparted on the interstitial fluid by the background circulation of the solid mantle matrix (Morgan, 1987; Spiegelman and McKenzie, 1987), a decompaction channel beneath the near surface mantle solidus (Sparks and Parmentier, 1991) and anisotropic permeability induced by the strain due to the mantle circulation (Morgan, 1987). Katz et al. (2006) noted that if melt bands form in the mantle below mid-ocean ridges and they are rotated by 25° from the direction of maximum compressive stress, as they are in the experiments and in the numerical simulations with highly strain-rate dependent viscosity, that they could act as a network of interconnecting high permeability pathways channeling melt towards the ridge axis.

One effect of buoyancy on a compacting porous medium is to induce oscillations and waves (e.g., Scott and Stevenson, 1986; Spiegelman, 1993). Where buoyancy-driven flow encounters a region of reduced permeability, fluid will build up leading to an increase in the porosity and permeability. As a result, more fluid will be drawn into this region, decreasing the porosity and permeability in the surrounding regions and resulting in propagating regions of increased and reduced porosity and permeability. It is the main purpose of this paper to investigate the interaction of this effect with strain-induced porosity localization. In agreement with Richardson (1998) we will show that bands can form in the presence of buoyancy and we will demonstrate that their growth rate is not affected by the degree of buoyancy. We will also show that strain-induced melt bands can channel flow in directions away from vertical. However, for highly strain-rate dependent viscosity there may be two different orientations of melt bands.

In what follows, the governing equations for the numerical simulations will first be presented. We will then present the linear theory of a compacting porous layer under the influence of an externally imposed simple shear and buoyancy when the matrix viscosity can be strain-rate dependent in Section 3 and we will compare some predictions of linear theory with the numerical model results. In Section 4, the results of numerical simulations with various degrees of strain-rate dependence of viscosity and buoyancy driven flow will be presented. Section 5 contains some interpretation and discussion of our results.

2. Governing equations

We solve the dimensionless equations appropriate for a compacting porous layer (e.g., McKenzie, 1984; Scott and Stevenson, 1984) in two space dimensions, x and y in a square domain of side length two compaction lengths centered on the origin. We impose a background, simple-shear, velocity of the solid matrix and fluid of the form $U_0 = y\,\sin(\theta)$, $V_0 = x\,\cos(\theta)$. Here θ can either be 0 or $\pi/2$ corresponding to shear in the vertical and horizontal directions. Our methodology can be easily generalized to other background flow geometries.

The dimensionless equations for the force balance of the fluid phase and for the solid matrix are

$$\phi(\mathbf{u}-\mathbf{U}) = -k_\phi(\nabla p - (1-\phi_0)B_u\hat{j}) \tag{1}$$

and

$$\nabla\cdot\begin{pmatrix} -p + \dfrac{\zeta+(4/3)\eta}{\zeta+(4/3)}\dfrac{\partial U}{\partial x} + \dfrac{\zeta-(2/3)\eta}{\zeta+(4/3)}\dfrac{\partial V}{\partial y} & \dfrac{\eta}{\zeta+(4/3)}\left(\dfrac{\partial U}{\partial y}+\dfrac{\partial V}{\partial x}+1\right) \\ \dfrac{\eta}{\zeta+(4/3)}\left(\dfrac{\partial U}{\partial y}+\dfrac{\partial V}{\partial x}+1\right) & -p + \dfrac{\zeta+(4/3)\eta}{\zeta+(4/3)}\dfrac{\partial V}{\partial y} + \dfrac{\zeta-(2/3)\eta}{\zeta+(4/3)}\dfrac{\partial U}{\partial x} \end{pmatrix} = \begin{pmatrix} 0 \\ (\phi_0-\phi)B_u \end{pmatrix}. \tag{2}$$

While equations indicating that the combined fluid and solid are incompressible and a mass conservation equation for the solid can be written

$$\nabla\cdot[\mathbf{u}\phi + \mathbf{U}(1-\phi)] = 0 \tag{3}$$

and

$$\frac{\partial\phi}{\partial t} = \nabla\cdot[(\mathbf{U} + y\,\sin(\theta)\hat{\imath} + x\,\cos(\theta)\hat{j})(1-\phi)]. \tag{4}$$

Here $\mathbf{u}$ and $\mathbf{U} = [U, V]$ represent velocity variations from the background flow for the fluid and solid phases, and p and ϕ are the transformed fluid pressure (see below) and the porosity while $\hat{\imath}$ and $\hat{j}$ are unit vectors in the horizontal and vertical directions. The equations are made dimensionless using scales for length, pressure, velocity and viscosity of δ_c, $\dot{\gamma}(\zeta_0 + 4/3\eta_0)$, $\dot{\gamma}\delta_c$ and η_0 where δ_c is the compaction length, $\dot{\gamma}$ is the strain rate corresponding to the background velocity and ζ_0 and η_0 are the dimensional bulk and shear viscosity of the matrix at the initial porosity and background strain. The parameters ζ and η represent the dimensionless bulk and shear viscosities. The compaction length is given by $\delta_c = (k_0(\zeta_0 + 4/3\eta_0)/\mu)^{0.5}$ where k_0 is the permeability at the initial porosity, and μ is the liquid viscosity. Both ζ and μ are assumed to be constant. Katz et al. (2006) reported simulations with and without a porosity dependence of the bulk viscosity and found very little resulting differences.

The transformed pressure, p, is related to the fluid pressure, p_{fluid}, by $p = p_{\mathrm{fluid}} + [(1-\phi_0)\rho_s + \phi_0\rho_l]B_u y$ where ρ_s and ρ_l are the dimensional solid and fluid densities divided by the difference between the solid and fluid densities and ϕ_0 is the initial background porosity. The dimensionless parameter B_u is defined in the following section. The pressure is transformed in this way so that the mean difference in p between the top and bottom boundaries is 0. This allows us to use periodic vertical boundary conditions.

The shear viscosity of the matrix is taken to weaken with porosity (Mei et al., 2002) and strain rate (Karato and Wu, 1993) according to

$$\eta = \exp(\alpha(\phi-\phi_0))\left[\sqrt{2}\left(\frac{\partial U}{\partial x}^2 + \frac{\partial V}{\partial y}^2 + 0.5\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}+1\right)^2\right)^{0.5}\right]^{((1-n_v)/n_v)}. \tag{5}$$

The parameter α is taken to be −25 (Mei et al., 2002) while the factor of $\sqrt{2}$ causes the dimensionless viscosity to be 1 when the velocity of the solid is equal to the background value. The value of the strain-rate exponent, n_v, is varied from one simulation to another from a minimum value of 1 (strain-rate-independent viscosity) to a maximum value of 6. Recently, Korenaga and Karato (2008) have