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Physics of the Earth and Planetary Interiors 165 (2007) 1–13
PHYSICS OF THE EARTH AND PLANETARY INTERIORS
www.elsevier.com/locate/pepi

Onset of convection in fluids with strongly temperature-dependent, power-law viscosity 2. Dependence on the initial perturbation

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Received 1 January 2007; received in revised form 4 May 2007; accepted 19 June 2007

Abstract

The onset of convection in the power-law creep regime on the silicate and icy planetary bodies requires a finite amplitude initial perturbation. This is a nonlinear problem and thus, both the amplitude and shape of the perturbation are important. We performed numerical simulations of the onset of convection in a two-dimensional layer with a fixed temperature contrast between the boundaries and in the stagnant lid regime of temperature-dependent viscosity convection. The optimal perturbations are located at the bottom of the layer (in the rheological sublayer) and have the wavelength of about twice the thickness of the rheological sublayer. The critical Rayleigh number for the onset of convection by optimal perturbations is calculated for a broad range of viscosity parameters. The results are summarized in terms of simple scaling relationships.

Keywords: Dislocation creep; Power-law viscosity; Onset of convection

1. Introduction

Constraints on the onset of convection in fluids with complicated rheologies are necessary to address long-standing questions about global convective stability of the interiors of terrestrial planets and icy satellites (Schubert et al., 1969; Reynolds and Cassen, 1979; McKinnon, 1999; Solomatov and Moresi, 2000; Barr et al., 2004; Barr and Pappalardo, 2005) as well as small-scale instabilities beneath continents and oceans on Earth (Jaupart and Parsons, 1985; Davaille and Jaupart, 1994; Lenardic and Moresi, 1999; Korenaga and Jordan, 2003; Huang et al., 2003; Sleep, 2005; Dumoulin et al., 2005) and at the base of the mantle (Yuen and Peltier, 1980; Solomatov and Moresi, 2002).

Although the onset of convection in constant viscosity fluids (Chandrasekhar, 1961) and in temperature-dependent viscosity fluids (Stengel et al., 1982; Richter et al., 1983; White, 1988) has been well studied, the onset of convection in power-law viscosity fluids, where dislocation creep and/or grain boundary sliding accommodate strain, is not well understood. Dislocation creep is a strong competitor to diffusion creep in planetary mantles and it is not easy to draw the boundary between the two. The dislocation creep regime may dominate convective instabilities in silicate bodies if one takes into account grain growth, which tends to suppress diffusion creep but has no effect on dislocation creep (Karato and Wu, 1993). In the terrestrial planets, convective instabilities at the bottom

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doi:10.1016/j.pepi.2007.06.007

Nomenclature

a	aspect ratio of the convective box
a_{cr}	critical aspect ratio in linear stability analysis
a^*_{cr}	aspect ratio at the critical point
b	pre-factor in the viscosity function
c_p	isobaric specific heat
d	depth of the convective box
d_{sub}	thickness of the rheological sublayer
E	activation energy
F	heat flux
g	acceleration due to gravity
k	thermal conductivity
L	width of the convecting box
n	stress exponent
Nu	Nusselt number
Nu^*_{cr}	Nusselt number at the critical point
R	gas constant
Ra	Rayleigh number
Ra_{cr}	critical Rayleigh number
$Ra_{cr,n}$	absolute minimum critical Rayleigh number for stress-dependent viscosity
$Ra_{cr,0}$	a constant in the scaling law
Ra_{sub}	Rayleigh number for the sublayer
Ra_δ	perturbation-dependent critical Rayleigh number
Ra^*_{cr}	absolute minimum critical Rayleigh number
$Ra^*_{cr,th}$	theoretically estimated absolute minimum critical Rayleigh number
t	time
T	temperature
ΔT	temperature contrast between the upper and lower boundaries
T_{cond}	temperature of a conductive layer
ΔT_{sub}	temperature contrast across rheological sublayer
T_0	surface temperature
T_1	bottom temperature
T_ξ	temperature field with imposed perturbation
u	velocity
x	horizontal coordinate
y	vertical coordinate
y_l	lower boundary of the sinusoidal perturbation
y_u	upper boundary of the sinusoidal perturbation
α	thermal expansion
γ	coefficient in the viscosity function
δT	temperature perturbation
δT_{cr}	critical perturbation for the onset of convection
δT_{sin}	sinusoidal temperature perturbation
δT_ξ	temperature perturbation constructed from the critical solution
δT^*_{cr}	amplitude of temperature variation at the critical point
$\delta T'_\xi$	temperature perturbation with zero horizontal average
δT_0	amplitude of sinusoidal temperature perturbation
$\Delta\eta$	viscosity contrast between the upper and lower boundaries
η	viscosity
θ	Frank–Kamenetskii parameter
κ	coefficient of thermal diffusivity
λ	perturbation wavelength
λ^*_{cr}	optimal wavelength
ξ	coefficient controlling the amplitude of temperature perturbation
τ	second invariant of deviatoric stress tensor
τ_T	stress due to initial perturbation

of the lithosphere are controlled by dislocation creep if the grain size exceeds ∼1 mm (Solomatov and Moresi, 2000). This is a possible situation because the grain size in the Earth's mantle varies from 0.1 mm to 1 cm (Karato and Wu, 1993). In icy satellites, convective instability may be governed by weakly non-Newtonian grain boundary sliding if ice grain sizes exceed 1 mm (Barr and Pappalardo, 2005; Barr and McKinnon, 2007).

The onset of convection in power-law viscosity fluids was investigated by Tien et al. (1969) and Ozoe and Churchill (1972). The onset of convection in the dislocation creep regime of Earth's materials was studied by Birger (1998, 2000). Birger's (1998, 2000) solutions describe the onset of convection for infinitesimal amplitudes, when the Andrade law is applicable (transient creep). However, even if the initial stage is described by the Andrade law, after about 10% strain the creep is in a steady-state regime (from the point of view of dislocation dynamics) and one has to investigate how the flow will evolve after that. Thus, one still needs to consider power-law viscosity.

Solomatov (1995) suggested an approximate equation for the onset of convection in the stagnant lid regime of temperature-dependent, power-law viscosity convection in a layer with a fixed temperature difference between the boundaries. In this regime, convection is