Onset of convection in fluids with strongly temperature-dependent, power-law viscosity

2. Dependence on the initial perturbation

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Received 1 January 2007; received in revised form 4 May 2007; accepted 19 June 2007

Abstract

The onset of convection in the power-law creep regime on the silicate and icy planetary bodies requires a finite amplitude initial perturbation. This is a nonlinear problem and thus, both the amplitude and shape of the perturbation are important. We performed numerical simulations of the onset of convection in a two-dimensional layer with a fixed temperature contrast between the boundaries and in the stagnant lid regime of temperature-dependent viscosity convection. The optimal perturbations are located at the bottom of the layer (in the rheological sublayer) and have the wavelength of about twice the thickness of the rheological sublayer. The critical Rayleigh number for the onset of convection by optimal perturbations is calculated for a broad range of viscosity parameters. The results are summarized in terms of simple scaling relationships.

Keywords: Dislocation creep; Power-law viscosity; Onset of convection

1. Introduction

Constraints on the onset of convection in fluids with complicated rheologies are necessary to address long-standing questions about global convective stability of the interiors of terrestrial planets and icy satellites (Schubert et al., 1969; Reynolds and Cassen, 1979; McKinnon, 1999; Solomatov and Moresi, 2000; Barr et al., 2004; Barr and Pappalardo, 2005) as well as small-scale instabilities beneath continents and oceans on Earth (Jaupart and Parsons, 1985; Davaille and Jaupart, 1994; Lenardic and Moresi, 1999; Korenaga and Jordan, 2003; Huang et al., 2003; Sleep, 2005; Dumoulin et al., 2005) and at the base of the mantle (Yuen and Peltier, 1980; Solomatov and Moresi, 2002).

Although the onset of convection in constant viscosity fluids (Chandrasekhar, 1961) and in temperature-dependent viscosity fluids (Stengel et al., 1982; Richter et al., 1983; White, 1988) has been well studied, the onset of convection in power-law viscosity fluids, where dislocation creep and/or grain boundary sliding accommodate strain, is not well understood. Dislocation creep is a strong competitor to diffusion creep in planetary mantles and it is not easy to draw the boundary between the two. The dislocation creep regime may dominate convective instabilities in silicate bodies if one takes into account grain growth, which tends to suppress diffusion creep but has no effect on dislocation creep (Karato and Wu, 1993). In the terrestrial planets, convective instabilities at the bottom

Nomenclature

\[ a \]
aspect ratio of the convective box

\[ a_{cr} \]
critical aspect ratio in linear stability analysis

\[ a^*_{cr} \]
aspect ratio at the critical point

\[ b \]
pre-factor in the viscosity function

\[ c_p \]
isobaric specific heat

\[ d \]
depth of the convective box

\[ d_{sub} \]
thickness of the rheological sublayer

\[ E \]
activation energy

\[ F \]
heat flux

\[ g \]
acceleration due to gravity

\[ k \]
thermal conductivity

\[ L \]
width of the convective box

\[ n \]
stress exponent

\[ Nu \]
Nusselt number

\[ Nu^*_{cr} \]
Nusselt number at the critical point

\[ R \]
gas constant

\[ Ra \]
Rayleigh number

\[ Ra_{cr} \]
critical Rayleigh number

\[ Ra_{cr,n} \]
absolute minimum critical Rayleigh number for stress-dependent viscosity

\[ Ra_{cr,0} \]
a constant in the scaling law

\[ Ra_{sub} \]
Rayleigh number for the sublayer

\[ Ra_{s} \]
perturbation-dependent critical Rayleigh number

\[ Ra^*_{cr} \]
absolute minimum critical Rayleigh number

\[ Ra^*_{cr,th} \]
theoretically estimated absolute minimum critical Rayleigh number

\[ t \]
time

\[ T \]
temperature

\[ \Delta T \]
temperature contrast between the upper and lower boundaries

\[ T_{cond} \]
temperature of a conductive layer

\[ \Delta T_{sub} \]
temperature contrast across rheological sublayer

\[ T_0 \]
surface temperature

\[ T_1 \]
bottom temperature

\[ T_\xi \]
temperature field with imposed perturbation

\[ u \]
velocity

\[ x \]
horizontal coordinate

\[ y \]
vertical coordinate

\[ y_l \]
lower boundary of the sinusoidal perturbation

\[ y_u \]
upper boundary of the sinusoidal perturbation

\[ \alpha \]
thermal expansion

\[ \gamma \]
coefficient in the viscosity function

\[ \delta T \]
temperature perturbation

\[ \delta T_{cr} \]
critical perturbation for the onset of convection

\[ \delta T_{sin} \]
sinusoidal temperature perturbation

\[ \delta T_\xi \]
temperature perturbation constructed from the critical solution

\[ \delta T^*_{cr} \]
amplitude of temperature variation at the critical point

\[ \delta T^*_{\xi} \]
temperature perturbation with zero horizontal average

\[ \delta T_0 \]
amplitude of sinusoidal temperature perturbation

\[ \Delta \eta \]
viscosity contrast between the upper and lower boundaries

\[ \eta \]
viscosity

\[ \theta \]
Frank–Kamenetskii parameter

\[ \kappa \]
coefficient of thermal diffusivity

\[ \lambda \]
perturbation wavelength

\[ \lambda^*_{cr} \]
optimal wavelength

\[ \xi \]
coefficient controlling the amplitude of temperature perturbation

\[ \tau \]
second invariant of deviatoric stress tensor

\[ \tau_T \]
stress due to initial perturbation

The onset of convection in power-law viscosity fluids was investigated by Tien et al. (1969) and Ozoe and Churchill (1972). The onset of convection in the dislocation creep regime of Earth’s materials was studied by Birger (1998, 2000). Birger’s (1998, 2000) solutions describe the onset of convection for infinitesimal amplitudes, when the Andrade law is applicable (transient creep). However, even if the initial stage is described by the Andrade law, after about 10% strain the creep is in a steady-state regime (from the point of view of dislocation dynamics) and one has to investigate how the flow will evolve after that. Thus, one still needs to consider power-law viscosity.

Solomatov (1995) suggested an approximate equation for the onset of convection in the stagnant lid regime of temperature-dependent, power-law viscosity convection in a layer with a fixed temperature difference between the boundaries. In this regime, convection is