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Two-dimensional direct current (DC) resistivity inversion: Data space Occam's approach

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ABSTRACT

A data space Occam's inversion algorithm for 2D DC resistivity data has been developed to seek the smoothest structure subject to an appropriate fit to the data. For traditional model space Gauss–Newton (GN) type inversion, the system of equations has the dimensions of $M \times M$, where M is the number of model parameter, resulting in extensive computing time and memory storage. However, the system of equations can be mathematically transformed to the data space, resulting in a dramatic drop in its dimensions to $N \times N$, where N is the number of data parameter, which is usually less than M. The transformation has helped to significantly reduce both computing time and memory storage. Numerical experiments with synthetic data and field data show that applying the data space technique to 2D DC resistivity data for various configurations is robust and accurate when compared with the results from the model space method and the commercial software RES2DINV.

1. Introduction

The direct current (DC) resistivity method has been used for various applications in hydrogeological, mining, and geotechnical investigations and environmental surveys (e.g., Ward, 1990; Daily et al., 1992, 1995; Ramirez et al., 1993, 1996; LaBrecque and Ward, 1990; among many others). The measured voltages caused by injected current bring out information on the earth's structure. The inversion program is then applied to interpret the measured voltages to obtain the Earth's resistivity structure.

The development of DC resistivity inversions has progressed successfully. Various techniques have been proposed for the two-dimensional (2D) and three-dimensional (3D) DC resistivity inversion (e.g., Pelton et al., 1978; Tripp et al., 1984; Nariida and Vozoff, 1984; Tong and Yang, 1990; Park and Van, 1991; Ellis and Oldenburg, 1994; Li and Oldenburg, 1994; Sasaki, 1994; Loke and Barker, 1995; Zhang et al., 1995; Loke and Dahlin, 1997, 2002; Tsourlos et al., 1998; Jackson et al., 2001; Pain et al., 2002; Loke et al., 2003; Günther et al., 2006; Pidlisecky et al., 2007; among many others). The most direct approach is the Gauss–Newton (GN) and its variant methods (e.g., Sasaki, 1994; Li and Oldenburg, 1994; Loke and Dahlin, 1997). Other limited memory optimization algorithms are the Quasi-Newton (QN) method (Loke and Barker, 1996; Loke and Dahlin, 1997, 2002; Tsourlos et al., 1998), the conjugate gradient (CG) type inversion (Zhang et al., 1995) and the non-linear conjugate gradient (NLCG) (Ellis and Oldenburg, 1994). These are the schemes that require the gradient of the function. The derivative-free methods are neural networks (El-Qady and Ushijima, 2001) and genetic algorithms (Schwarzbach et al., 2005).

One of the main disadvantages of the GN-type inversion is that it requires solving a large and dense $M \times M$ system of equations, where M is the number of model parameters. Another disadvantage is the formation of the full $N \times M$ Jacobian or sensitivity matrix. Calculation of the full Jacobian requires a numerical solution of many forward problems. Both disadvantages, consequently, result in extensive computing time and memory storage. For example, in the 3D inversion, the synthetic model of a burial mound and data used by Günther et al. (2006) has 23,109 parameter cells (M = 23,109) which is a lot more than the number of data parameters (N = 3439). Inverting the 23,109 × 23,109 matrix and forming the Jacobian would require about 4–5 GBytes of RAM and many hours of CPU time.

The problem for the 3D DC resistivity inversion is quite similar to (though not as severe as) that for the 3D magnetotelluric (MT) survey, where the model parameter (M) is significantly greater than the data parameter (N). Siripunvaraporn and Egbert (2000) and Siripunvaraporn et al. (2005) could overcome this difficulty by transforming the model space inverse problem into the data space problem for their 2D and 3D Magnetotelluric data, respectively. With the transformation, the computational time and memory storage are greatly reduced by a factor of several (Siripunvaraporn and

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Egbert, 2000; Siripunvaraporn et al., 2005). However, formation of the Jacobian matrix is still a requirement.

For the limited memory optimization schemes such as QN, the full Jacobian or sensitivity matrix and the large and dense coefficient matrix of the system of equations are not necessarily constructed. Instead, a multiplication of the Jacobian with any vector can be calculated by solving the forward problem. These methods therefore never require a large memory storage as in GN-type inversions. Another advantage of QN-type inversions over the model space GN-type is their speed. However, their stability may be questionable (Loke and Dahlin, 2002). Though GN-type inversions may use extensive computing time, their main advantages are stability and robustness. They require fewer iterations to converge to the solution than limited memory methods (Loke and Dahlin, 2002; Siripunvaraporn and Egbert, 2007).

Because of their stability, we still have confidence in GN-type inversion techniques, especially Occam's method as first introduced by Constable et al. (1987). Siripunvaraporn and Egbert (2007) showed that for 2D MT data, the computing time of a GN-type inversion in the data space is actually comparable to that of the CG or NLCG inversion. For all of these reasons, here we propose to solve the multi-dimensional DC resistivity inverse problem using one variant GN-technique, Occam's inversion. However, instead of solving the problem in model space as others have (e.g., Constable et al., 1987; Sasaki, 1994), we propose to solve the DC resistivity inverse problem in data space as in Siripunvaraporn and Egbert (2000) and Siripunvaraporn et al. (2004, 2005). In order to test the feasibility and practicality of the data space approach for 3D DC resistivity data, we developed the 2D DC resistivity inversion based on the data space approach of Siripunvaraporn et al. (2005), which will be extended to 3D in the future.

We first start the paper by briefly reviewing the basic idea of Occam's inversion in the usual model space formulation, and then from a data space perspective. We then describe the implementation of the data space technique to a 2D DC resistivity data set. Numerical experiments of both synthetic and real field data in comparison with the commercial software RES2DINV version 3.55 (Loke and Barker, 1996) are shown at the end.

2. Occam's inversion: model space approach versus data space approach

Constable et al. (1987) introduced the Occam method for 1D MT and Schlumberger sounding data. Since then it has become one of the "classic" inversion techniques for various geophysical data (e.g., deGroot-Hedlin and Constable, 1990, 2004; deGroot-Hedlin, 1995; LaBrecque et al., 1996; Siripunvaraporn and Egbert, 2000; Huang et al., 2003; Siripunvaraporn et al., 2005; Greenhalgh et al., 2006; among others). For more general and detailed discussions of the Occam approach, see Constable et al. (1987), deGroot-Hedlin and Constable (1990), Siripunvaraporn and Egbert (2000) and Siripunvaraporn et al. (2004, 2005).

The philosophy of the Occam approach is to seek for the "smoothest" or "minimum" structure model subject to a constraint on the misfit (Constable et al., 1987), which can be mathematically translated into a problem of minimization of an unconstrained functional $U(\mathbf{m}, \lambda)$,

$$U(\mathbf{m}, \lambda) = (\mathbf{m} - \mathbf{m_0})^{\mathrm{T}}\mathbf{C_m^{-1}}(\mathbf{m} - \mathbf{m_0}) + \lambda^{-1}\{(\mathbf{d} - \mathbf{F}[\mathbf{m}])^{\mathrm{T}}\mathbf{C_d^{-1}}(\mathbf{d} - \mathbf{F}[\mathbf{m}]) - X^{*2}\}. \quad (1)$$

Here $\mathbf{m}$ is a resistivity or conductivity model of dimension M, $\mathbf{m_0}$ a base or prior model, $\mathbf{C_m}$ a model covariance matrix which defines the model norm, $\mathbf{d}$ the observed data with dimension N, $\mathbf{F}[\mathbf{m}]$ the forward model response, $\mathbf{C_d}$ a data covariance matrix, X^* the desired level of misfit, and λ^{-1} a Lagrange multiplier. In the 2D DC resistivity case, the data $\mathbf{d}$ are the apparent resistivities from different configurations. The model response $\mathbf{F}[\mathbf{m}]$ is computed by solving the DC resistivity forward problem, which we will describe later.

Instead of directly minimizing (1), Constable et al. (1987) consider the penalty functional $W_\lambda(\mathbf{m})$,

$$W_\lambda(\mathbf{m}) = (\mathbf{m} - \mathbf{m_0})^{\mathrm{T}}\mathbf{C_m^{-1}}(\mathbf{m} - \mathbf{m_0}) + \lambda^{-1}\{(\mathbf{d} - \mathbf{F}[\mathbf{m}])^{\mathrm{T}}\mathbf{C_d^{-1}}(\mathbf{d} - \mathbf{F}[\mathbf{m}])\}. \quad (2)$$

When λ is fixed, $\partial U/\partial m$ and $\partial W_\lambda/\partial m$ yield the same result. Therefore, minimizing W_λ with a series of λ values, and choosing λ for which the smallest minimum is achieved, is equivalent to minimizing the original functional U of (1).

Because of the non-linearity of the inverse problem, the Occam's inversion starts with the linearization of the response function $\mathbf{F}[\mathbf{m}]$ based on the Taylor series expansion, $\mathbf{F}[\mathbf{m}_{k+1}] = \mathbf{F}[\mathbf{m}_k] + \mathbf{J}_k(\mathbf{m}_{k+1} - \mathbf{m}_k)$. Inserting the series expansion in (2), and then solving for the stationary points, a series of iterative approximate solutions is then obtained,

$$\mathbf{m}_{k+1}(\lambda) - \mathbf{m_0} = [\lambda\mathbf{C_m^{-1}} + \mathbf{J}_k^{\mathrm{T}}\mathbf{C_d^{-1}}\mathbf{J}_k]^{-1}\mathbf{J}_k^{\mathrm{T}}\mathbf{C_d^{-1}}\hat{\mathbf{d}}_k, \quad (3)$$

where $\hat{\mathbf{d}} = \mathbf{d} - \mathbf{F}[\mathbf{m}_k] + \mathbf{J}_k(\mathbf{m}_k - \mathbf{m}_0)$, the subscript k denotes the iteration number, and $\mathbf{J}_k = (\partial\mathbf{F}/\partial\mathbf{m})_k$ is the $N \times M$ sensitivity or Jacobian matrix calculated at $\mathbf{m}_k$. Note that the system of Eq. (3) has dimensions of $M \times M$. We therefore called this technique the "model space" Occam's inversion.

Parker (1994) showed that the solution to (3) for iteration k can be transformed to

$$\mathbf{m}_{k+1} - \mathbf{m_0} = \mathbf{C_m}\mathbf{J}_k^{\mathrm{T}}\boldsymbol{\beta}_{k+1}, \quad (4)$$

where $\boldsymbol{\beta}_{k+1}$ is an unknown expansion coefficient vector. The derivation of (4) from (3) is also given in Siripunvaraporn et al. (2005). Searching for the stationary points with the transformation (4), a series of iterative solutions is again obtained,

$$\boldsymbol{\beta}_{k+1} = [\lambda\mathbf{C_d} + \mathbf{J}_k\mathbf{C_m}\mathbf{J}_k^{\mathrm{T}}]^{-1}\hat{\mathbf{d}}_k. \quad (5)$$

Note that the system of Eq. (5) has dimensions $N \times N$, rather than $M \times M$ as in (3). Here is the main difference between (3) and (5). Because we transform the computation from model space to data space, we therefore called this technique after the transformation the "data space" Occam's inversion. If all the same parameters are used the solutions from both approaches will be identical (Siripunvaraporn and Egbert, 2000; Siripunvaraporn et al., 2005). For MT data, the number of model parameters M is usually much larger than the number of data values N. Both the calculation time and memory are significantly decreased with the transformation to data space (Siripunvaraporn and Egbert, 2000; Siripunvaraporn et al., 2005). Here, we apply this method to DC resistivity data and we expect to gain the same benefits.

The beauty of Occam's inversion is here, which makes it different from other regularized inverse problems. In either the model space or data space approach, the goal is to search for the minimization of (1). This can be performed by two stages. The first stage (Phase I) is to bring the misfit down to the target level by varying λ values in (3) and (5) for each iteration. Once the target misfit is achieved, Phase II keeps the misfit at the desired level and searches for the minimum norm model by again varying λ values in each iteration. The addition of Phase II is to guarantee that the model structure does not contain unwanted or spurious structures (Siripunvaraporn et al., 2004, 2005).