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Establishing performance evaluation structures by fuzzy relation-based cluster analysis

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Abstract

The purpose of this study is to establish hierarchical structures for the performance evaluation of vague, humanistic complicated systems. To overcome the difficulties due to partial information and the vagueness of human knowledge and recognition, a fuzzy relation-based clustering method is proposed to model this evaluation. First, the effects of different max-t_i compositions on the formation of clusters are discussed. Then, an improved clustering algorithm is developed to produce several partition trees with different levels and clusters according to different t_i-norm compositions. To demonstrate the usefulness of the proposed algorithm, the academic departments of higher education were considered using actual engineering school data in Taiwan. Three performance evaluation structures are established by using max-t_1, max-t_2 and max-t_3 compositions. The results show that the proposed fuzzy hierarchical approach is useful and practical for performance evaluations of complicated humanistic systems.

Keywords: Fuzzy sets; Fuzzy cluster analysis; Fuzzy relation; Max-t_i composition; t_i-norm; Performance evaluation; Hierarchical evaluation structure

1. Introduction

In the field of performance evaluation, how to establish an effective model for objective assessment has been a continuous concern for decision-makers (DMs). The problem of evaluation analysis, particularly for large-scale complex processes, is usually represented, as a hierarchy of goals and means, in the shape of a systematic diagram composed of all the criteria elements. In other words, a hierarchical structure is formed. Fogliatto and Albin [1] and Salo and Punkka [2] stated that based on the multiple-criterion and multiple-level types of assessment structure, it is easier to judge the relative importance of the criteria. Moreover, the hierarchical evaluation structure can be

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easily applied to real-world problems. Stillwell et al. [3] found that the hierarchical weights are better than the non-hierarchical weights in providing a more powerful method to identify the performance differences among groups of evaluated objects.

Due to the above-mentioned advantages of using a hierarchical structure, various methods for performance evaluation, such as analytical hierarchy process (AHP) [4–9] and hierarchy consistency analysis (HCA) [10–14], have been developed. A common fundamental problem for these methods is to determine the schema of a hierarchical evaluation structure, which critically influences the correctness of the evaluation results. Studies have been made specifically on this issue. In most cases, the decision-maker, to intuitively emphasize some specified evaluation viewpoints, roughly constructs a hierarchical evaluation structure. At the same time one must still consider whether the employed hierarchical evaluation structure is appropriate, and if the structure is biased due to the DM's prejudice and/or incorrect viewpoint.

The problem of establishing a hierarchical evaluation structure can be considered as a clustering process to group the related various criteria together. However, due to the availability and uncertainty of information, and the vagueness of human feelings and recognition, a fuzzy clustering method should be used. Another important point to consider is the actual application aspects. Since the implementation cost goes up in direct proportion to the accuracy of the model, different degrees of precisions should be considered. Thus, it is desirable to establish a mechanism that can produce various different schemas, from the rough, with few clusters and few levels, to the more detailed, with more clusters and more levels. To achieve this purpose, the t_i-norm [15] fuzzy compositions are used to form groups of related criteria. Furthermore, the clustering algorithm due to Yang and Shih [16] is improved and used to build several hierarchical evaluation structures corresponding to different max-t_i compositions.

The rest of the paper is organized as follows. In Section 2, the basic concept of fuzzy relation analysis is introduced based on max-t_i compositions. In Section 3, the theory of establishing a max-t_i-similarity relation matrix and the process to derive a clustering as a hierarchical evaluation structure are first introduced. Then, a clustering algorithm was developed to improve the algorithm due to Yang and Shih's algorithm [16] so that it can yield a partition tree with an "inclusive relation" between the partitions of adjacent R_α for any max-t_i-similarity relation matrix. In Section 4, the proposed clustering algorithm is applied to an actual example obtained from the Taiwan Assessment and Evaluation Association (TWAEA) concerning the evaluation of the performance of the academic departments of higher education in Taiwan. Three performance evaluation structures are established by using max-t_1, max-t_2 and max-t_3 compositions. Finally, some general conclusions are given in Section 5.

2. Fuzzy relation-based cluster analysis

Cluster analysis, a tool for data analysis, is a branch in statistical multivariate analysis and also is an unsupervised learning technique in pattern recognition. Since Bellman et al. [17] and Ruspini [18] first initiated the research on clustering based on fuzzy sets [19], fuzzy clustering has been widely studied and applied in a variety of different areas (see Bezdek [20], Yang [21], Hoppner et al. [22]). These fuzzy clustering methods can be roughly divided into two categories. One involves distance-defined objective functions. The fuzzy c-means (FCM) algorithm and its variations are the well-known approaches in this category (see Baraldi and Blonda [23], Lee [24], Yu and Yang [25]). However, the FCM-type methods need to have data presented in feature vectors so that the distance and prototypes can be calculated.

The other category involves fuzzy relations. Since these fuzzy relation-based methods require only a relation matrix of the data set, they are simpler to use in some applications (see Tamura et al. [26], Dunn [27], Yang and Shih [16]). The approach can use fuzzy values to represent the degree of similarity between two objects and can be applied to many areas such as data mining, web mining and database acquisition. This category of fuzzy clustering was originally developed for obtaining an agglomerative hierarchical clustering. Thus, the approach is especially suited for establishing a hierarchical schema of performance evaluation.

Let a crisp (binary) relation R between two sets, X and Y, be defined as a subset of $X \times Y$. Denoted by $R(X, Y)$, this relation is associated with an indicator function $u_R(x, y)$ which belongs to {0, 1} for all $(x; y)$ in $X \times Y$. That is, $u_R(x, y) = 1$ if $(x, y) \in R(X, Y)$, and $u_R(x, y) = 0$ if $(x, y) \notin R(X, Y)$. To avoid this yes or no restriction, Zadeh [28] defined a fuzzy relation R between X and Y as a fuzzy subset of $X \times Y$ by an extension of allowing $u_R(x, y)$ being a membership function assuming values in the interval [0, 1]. The value of $u_R(x, y)$ represents the strength of the relationship between x and y. The t-norm has been defined as a general form of a fuzzy intersection