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A novel description of fluid flow in porous and debris materials

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A B S T R A C T

Based on a generalized two-phase mass flow model (Pudasaini, 2012) as a mixture of solid particles and interstitial fluid, here, we derive a novel dynamical model equation for sub-diffusive and sub-advective fluid flow in general porous media and debris material in which the solid matrix is stationary. We construct some exact analytical solutions to the new model. The complete exact solutions are derived for the full sub-diffusive fluid flows. Solutions for the classical linear diffusion and the new sub-diffusion with quadratic fluxes are compared, and the similarities and differences are discussed. We show that the solution to sub-diffusive fluid flow in porous and debris material is fundamentally different from the diffusive fluid flow. In the sub-diffusive process, the fluid diffuses slowly in time, and thus, the flow (substance) is less spread. Furthermore, we construct some analytical solutions for the full sub-diffusion and sub-advection equation by transforming it into classical diffusion and advection structure. High resolution numerical solutions are presented for the full sub-diffusion and sub-advection model, which is then compared with the solution of the classical diffusion and advection model. Solutions to the sub-diffusion and sub-advection model reveal very special flow behavior, namely, the evolution of forward advecting frontal bore head followed by a gradually thinning tail that stretches to the original rear position of the fluid. However, for the classical diffusion–advection model, the fluid simply advects and diffuses. Moreover, the full sub-diffusion and sub-advection model solutions are presented both for the linear and quadratic drags, which show that the generalized drag plays an important role in generating special form and propagation speed of the sub-diffusion–advection waves. We also show that the long time solution to sub-diffusive and sub-advective fluid flow through porous media is largely independent of the initial fluid profile. These exact, analytical and numerical solutions reveal many essential physical phenomena, and thus may find applications in modeling and simulation of environmental, engineering and industrial fluid flows through general porous media and debris materials.

1. Introduction

Macroscopic averaged equations describing fluid flow through porous media are of great practical and theoretical interest in science and technology including oil exploration and environment related problems (Darcy, 1856; Richards, 1931; Bear, 1972; De Marsily, 1986; Durlofsky and Brady, 1987; Dagan, 1989). There are several geophysical and industrial applications of such fluid flows. This includes the flow of liquid or gas through soil and rock (e.g., shell oil extraction), clay, gravel and sand, or through sponge and foam. From geophysical and engineering prospectives, the fluid flows through porous and debris media are important aspects as it is coupled with the stability of the slope, the subsurface hydrology, and the transportation of chemical substances in porous landscape. Proper understanding of fluid flows in debris material and porous landscape, and in general through porous media, is an important aspect in industrial applications, geotechnical engineering, engineering geology, subsurface hydrology and natural hazard related phenomena (Muskat, 1937; Barenblatt, 1952; Bear, 1972; Whitaker, 1986; Boon and Lutsko, 2007; Vazquez, 2007). Better and reliable understanding of slope stability analysis, landslide initiation, debris and avalanche deposition morphologies, including seepage of fluid through relatively stationary porous matrix and consolidation, require more accurate and advanced knowledge of fluid flows in porous materials. Understanding the dynamics of fluid flows in porous landscape may help to develop early warning strategies in potentially huge and catastrophic failure of landslides, reservoir dams and embankments in geo-disaster-prone areas (Genevois and Ghirotti, 2005; Pudasaini and Hutter, 2007; Khattri, 2014; Miao et al., 2014; Pudasaini, 2014), and deposition processes of subsequent mass flows (Zhang et al., 2011; Kuo et al., 2011; Mergili et al., 2012; Tai and Kuo, 2012; Fischer, 2013; Wang et al., 2013; Zhang and Yin, 2013; Yang et al., 2015). Here, the terms porous landscape, debris material and porous media are used as synonym, because in all these materials we assume that the fluid passes through the relatively stationary solid skeleton, or matrix of granular particles.

Classically, the flow of a fluid through a homogeneous porous medium is described by the porous medium equation. The model is derived by using the continuity equation for the flow of ideal fluid through porous medium, the Darcy law relating fluid pressure gradient to the mean velocity, and by assuming a state equation for ideal fluid in

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which pressure is an explicit exponent (≥1) function of the fluid (gas) density. The resulting porous media equation is similar in form to the classical diffusion equation, except that, in the porous medium equation, the diffusive flux is non-linear with exponent ≥2 (Bear, 1972; Smyth and Hill, 1988; Vazquez, 2007) which is ultimately responsible for the slow diffusion process. Exact solutions exist for the porous medium equation (Barenblatt, 1952, 1953; Evans, 2010; Boon and Lutsko, 2007) in similarity form. Unlike the classical diffusion equation, the solution for porous medium equation is bounded by finite propagation speed of the flow fronts, and a compact support (Daskalopoulos, 2009), which means the solution is contained within a fixed domain.

Based on the generalized two-phase mass flow model (Pudasaini, 2012) this paper presents a novel sub-advection and sub-diffusion model equation and its analytical solutions for fluid flows through inclined debris materials and porous landscapes with stationary solid matrix. In these scenarios, we show that a number of important physical phenomena are rather governed by the nonlinear advection and diffusion processes that are associated with exact description of physical model parameters. Our results reveal that solutions to the sub-diffusive fluid flow in porous media and porous landscape is fundamentally different from the classical diffusive fluid flow or diffusion of heat, tracer particles and pollutant in fluid. Reduced models and exact solutions are presented for the sub-diffusive fluid flow. We outline some possible and systematic ways to construct analytical solutions for our new full sub-diffusion and sub-advection equation (also see, Boon and Lutsko, 2007). This includes the transformation of the model (in the form) to the convenient classical diffusion–advection equation for which we have constructed advanced analytical solutions by using the Bring ultraradical (Bring, 1864) and higher-order hypergeometric function. Furthermore, separation of variables leads to special ordinary differential equations in the form of Lienard and Abel canonical equations, that may provide other set of exact solutions (Lienard, 1928). We have presented numerical solution to such model that provides insight into the intrinsic nature of fluid flow in porous media.

New analytical solutions for fluid flows through debris material and porous landscape are then compared with numerical simulations to measure the performance of the numerical method for their further use in relevant fluid flows. The widely used high-resolution, shock-capturing Total Variation Diminishing Non-oscillatory Central (TVD-NOC) scheme (Nessyahu and Tadmor, 1990; Tai et al., 2002; Pudasaini, 2011) is implemented to solve the model equations numerically. For alternative numerical methods including finite volume, discrete element methods, spring-deformable-block model, we refer to Crosta et al. (2003), Mangeney-Castelnau et al. (2003), Denlinger and Iverson (2004), Pirulli (2009), Teufelsbauer et al. (2011) and Yang et al. (2015). Numerical results are presented for the full sub-diffusion and sub-advection model, which is then compared with the solution of the classical diffusion and advection model. Special features associated with the new model, and as revealed by the exact, analytical, and numerical solutions, are discussed in detail. Moreover, the full sub-diffusion and sub-advection model solutions are presented both for the linear and quadratic drags. The long time solutions are analyzed for different initial fluid profiles.

On the one hand, exact, analytical, and numerical solutions disclose many new and essential physics, and thus, may find applications in environmental, engineering and industrial fluid flows through general porous media, natural slopes, embankments (e.g., of hydro-electric power reservoirs and dams), and debris materials. While on the other hand, analytical and exact solutions to simplified cases of nonlinear model equations are necessary to calibrate numerical solution methods (Pudasaini, 2011). The reduced and problem-specific solutions provide important insights into the full behavior of the complex two-phase system, mainly the flow of fluid through the porous media. Broadly speaking, these results can further be applied to the problems related to hydrogeology, and environmental pollution remediation.

2. The two-phase mixture model

In order to develop a new sub-diffusion and sub-advection model for fluid flow through a porous media, we consider the general two-phase mass flow model (Pudasaini, 2012) that describes the dynamics of a real two-phase debris flow as a mixture of the solid particles and the interstitial fluid. The model is developed within the framework of continuum mechanics. For more on multi-phase and other relevant flows, we refer to Richardson and Zaki (1954), Anderson and Jackson (1967), Drew (1983), Ishii and Hibiki (2006), and Kolev (2007). The two phases are characterized by distinct material properties: the fluid phase is characterized by its true density ρ_f, viscosity η_f, and isotropic stress distribution, whereas the solid phase is characterized by its material density ρ_s, internal and basal friction angles, ϕ and δ, respectively, and an anisotropic stress distribution, K (lateral earth pressure coefficient). These characterizations and the presence of relative motion between these phases result in two different mass and momentum balance equations for the solid and the fluid phases, respectively. Let u_s, u_f and α_s, α_f $(=1\text{-}\alpha_s)$ denote the velocities, and volume fractions for the solid and the fluid constituents, denoted by the suffices s and f, respectively. The general two-phase debris flow model reduced to one-dimensional flows down a slope are described by the following set of non-linear partial differential equations (Pudasaini, 2012, 2014; Pudasaini and Miller, 2012; Pudasaini and Krautblatter, 2014):

$$\frac{\partial}{\partial t}(\alpha_s h) + \frac{\partial}{\partial x}(\alpha_s h u_s) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\alpha_f h) + \frac{\partial}{\partial x}(\alpha_f h u_f) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}\left[\alpha_s h(u_s - \gamma\mathcal{C}(u_f - u_s))\right] + \frac{\partial}{\partial x}\left[a_s h\left(u_s^2 - \gamma\mathcal{C}\left(u_f^2 - u_s^2\right) + \frac{\beta_s h}{2}\right)\right] = hS_s, \tag{3}$$

$$\frac{\partial}{\partial t}\left[\alpha_f h\left(u_f + \frac{\alpha_s}{\alpha_f}\mathcal{C}(u_f - u_s)\right)\right] + \frac{\partial}{\partial x}\left[a_f h\left(u_f^2 + \frac{\alpha_s}{\alpha_f}\mathcal{C}\left(u_f^2 - u_s^2\right) + \frac{\beta_f h}{2}\right)\right] = hS_f. \tag{4}$$

Eqs. (1) and (2) are the depth-averaged mass balances, and Eqs. (3) and (4) are the depth-averaged momentum balance equations for the solid and the fluid phases, respectively.

The force/source terms in the momentum equation for the solid-phase (Eq. (3)) is

$$\begin{aligned} S_s = \alpha_s\left[g^x - \frac{u_s}{|u_s|}\tan\delta p_{b_s} - \varepsilon p_{b_s}\frac{\partial b}{\partial x}\right] - \varepsilon\alpha_s\gamma p_{b_f}\left[\frac{\partial h}{\partial x} + \frac{\partial b}{\partial x}\right] \\ + C_{DG}(u_f - u_s)|u_f - u_s|^{j-1}. \end{aligned} \tag{5}$$

Similarly, the force/source term for the fluid-phase (Eq. (4)) is

$$\begin{aligned} S_f = \alpha_f\Bigg[g^x - \varepsilon\Bigg[\frac{1}{2}p_{bf}\frac{h}{\alpha_f}\frac{\partial\alpha_s}{\partial x} + p_{bf}\frac{\partial b}{\partial x} - \frac{1}{\alpha_f N_R}\left\{2\frac{\partial^2 u_f}{\partial x^2} - \frac{\chi u_f}{\varepsilon^2 h^2}\right\} \\ + \frac{1}{\alpha_f N_{R_\mathcal{A}}}\left\{2\frac{\partial}{\partial_x}\left(\frac{\partial\alpha_s}{\partial x}(u_f - u_s)\right)\right\} - \frac{\xi\alpha_s(u_f - u_s)}{\varepsilon^2\alpha_f N_{R_\mathcal{A}} h^2}\Bigg]\Bigg] - \frac{1}{\gamma}C_{DG}(u_f - u_s)|u_f - u_s|^{j-1}. \end{aligned} \tag{6}$$

In Eqs. (5) and (6), $j=1$ or, 2 correspond to the linear, or quadratic drags. The other parameters are

$$\begin{aligned} &\beta_s = \varepsilon K p_{b_s}, \quad \beta_f = \varepsilon p_{b_f}, \quad p_{b_f} = -g^z, \quad p_{b_s} = (1-\gamma)p_{b_f}, \\ &C_{DG} = \frac{\alpha_s\alpha_f(1-\gamma)}{\left[\varepsilon\,\mathcal{U}_T\{\mathcal{P}\mathcal{F}(Re_p) + (1-\mathcal{P})\mathcal{G}(Re_p)\}\right]^j}, \quad \mathcal{F} = \frac{\gamma}{180}\left(\frac{\alpha_f}{\alpha_s}\right)^3 Re_p, \quad \mathcal{G} = \alpha_f^{M(Re_p)-1}, \\ &\gamma = \frac{\rho_f}{\rho_s}, \quad Re_p = \frac{\rho_f d\,\mathcal{U}_T}{\eta_f}, N_R = \frac{\sqrt{gL}H\rho_f}{\alpha_f\eta_f}, \quad N_{R_\mathcal{A}} = \frac{\sqrt{gL}H\rho_f}{\mathcal{A}\eta_f}, \quad \alpha_f = 1-\alpha_s. \end{aligned} \tag{7}$$

In the above equations, t is the time, h is the flow depth (or, porous material height). p_{b_f} and p_{b_s} are associated with the effective fluid and solid pressures. x and z are coordinates along the flow directions, and