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Technical note

Estimating the elastic moduli and isotropy of block in matrix (bim) rocks by computational homogenization

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ABSTRACT

Block-in-matrix (bim) rocks are comprised of stiff inclusions (blocks) in softer matrix. The heterogeneity and multi-scale nature of bim-rocks makes the evaluation of their mechanical properties a challenging task. In this research we study the elastic moduli (E, ν, K. μ) and isotropy of bim-rocks using finite elements based computational homogenization and multi-scale approach. The influence of volumetric block proportion (VBP), block shape and block orientation is presented. Application of multi-scale homogenization method to estimate the elastic moduli of a naturally occurring conglomerate is also reported. When compared to close form solution of Hashin–Shtrikman the computational homogenization approach shows good agreement for bim-rocks with spherical blocks. For bim-rocks with elliptical blocks the elastic moduli show tangible difference from the closed form solution, with deviation from isotropy increasing with VBP and preferred orientation of blocks. Based on the results of our numerical study we propose a simplified approximation for estimation of elastic moduli of bim-rocks, together with estimates of deviation from isotropy.

1. Introduction

Various geological materials are comprised of stiff inclusions embedded in softer matrix, e.g. conglomerates, agglomerates, brecias (volcanic or tectonic), shear zones and mélanges, among other rock types. These rocks are considered problematic in most geo-engineering projects due to their spatial and compositional variability. The term block in matrix (bim) rocks was introduced by Raymond (1984) and abbreviated to "bim-rocks" by Medley (1994). The definition of a bim-rock, according to Medley, is "a mixture of rocks, composed of geotechnically significant blocks within a bonded matrix of finer texture".

Similarly to other rock/soil mixtures the size distribution of bim-rocks tends to be fractal (or well graded in geotechnical notation). Medley and Lindquist (1995) argued that this distribution is scale independent, as blocks are always found, regardless of the scale of interest or observation. They have defined block–matrix threshold of $0.05L_c$, where L_c is the characteristic length of blocks in the assemblage. Mechanical contrast between blocks and matrix is typically defined using shear strength ratio (friction angles) or stiffness ratio. Sufficient contrast is afforded by a friction angle ratio (tan φ of weakest block)/(tan φ of matrix) of between 1.5 and 2, as suggested by the works of Lindquist (Lindquist, 1994b; Lindquist and Goodman, 1994) and Volpe et al. (1991). Another means of identifying strength contrasts is to use rock stiffness. Lindquist (1994a) used a ratio of block to matrix stiffness (E_b/E_m), of 2.0 to generate block/matrix contrasts matrix for physical models of mélange. Similar, but not exclusive values for stiffness ratios were also reported by other researchers (e.g. Afifipour and Moarefvand, 2014; Sonmez et al., 2006b, 2004).

The overall mechanical properties of bim-rocks are affected by the mechanical properties of the matrix and the blocks, the volumetric block proportion (VBP), the block shapes, the block size distributions and the orientation of the blocks. Different techniques to determine the VBP from outcrop and boring data were reported in the literature, from simple scan lines to state of the art image processing techniques (Coli et al., 2012; Xu et al., 2008).

Many researchers have shown that the VBP is the single most important parameter determining the physical and mechanical properties of bim-rocks (e.g. Afifipour and Moarefvand, 2014; Coli et al., 2012; Kahraman et al., 2015; Lindquist and Goodman, 1994; Sonmez et al., 2006a, 2004).

When the VBP ranges from 0.25 and 0.7, the increase in the overall mechanical properties of bim-rocks are directly related to the volumetric block proportion of blocks in the rock mass (Lindquist and Goodman, 1994). For VBP > 0.7 the rock mass cannot be treated as a bim-rock but rather as "blocky rock mass with infilled joints" (Medley, 1994), as blocks tend to develop "contact to contact" geometry. Recently Kahraman et al. (2015) showed linear correlation between VBP and pressure wave velocity for VBP range from 0.02 to 0.9 for the Misis fault breccia.

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Many mechanical problems in geo-engineering are analyzed within the framework of elasto-plasticity, for which the elastic constants (i.e. E, ν, K, and μ) of the rock material and the rock mass are essential parameters. In many cases homogeneity and isotropy are also assumed. While these assumptions may be safely made for continuous, fine-grained rocks, they cannot be assured for bim-rocks due to their inherent nature, a mix of different constituents with distinct mechanical properties and variable proportions. As pointed out by Sonmez et al. (2006b) in many rock-engineering applications, the elastic modulus of intact rock is not actually determined by laboratory tests—due to the requirements of high-quality core samples and sophisticated test equipment. Moreover it is sometimes too difficult to obtain standard core samples from weak, stratified, highly fractured and block-in-matrix rocks.

Laboratory determination of the bim-rocks elastic moduli is difficult due to the variability of block size distribution in an outcrop, which in many cases exceeds the standard sample sizes, such as the ISRM suggested NX size (Ulusay and Hudson, 2006). Lindquist and Goodman (1994) proposed the use of laboratory scale models in order to determine the strength and deformability of mélanges. Their main reasoning was the scale invariability of the block distribution of the mélanges. This approach however is not always practical, as fabrication of physical models and their testing are time consuming. A large number of specimens should be prepared to ensure reproducibility, and moreover to test various stress paths and loading configurations. The physical model should also conform to the mechanical properties of blocks, in many cases of various origins and rock types, and matrix.

Sonmez et al. (2006a) used large diameter (150 mm) cores to study the uniaxial compressive strength (UCS) of the Ankara agglomerate as a complementary measure to studies based on statistical evaluation of the elastic constants and strength of bim-rock properties (Sonmez et al., 2006b, 2004). Wen-Jie et al. (2011) and Coli et al. (2011) performed unconventional large scale shear tests on soil rock mixture (SRM) and Shale–Limestone Chaotic Complex bim-rock (SLCC) respectively. These tests focused on shear strength rather than elastic properties. In general the main findings are that for VBP < 0.25 the unconfined compressive strength (UCS) reflects the properties of the matrix, whereas for VBP > 0.7 UCS reflects the properties of the blocks. UCS for VBP in the range of 0.25 to 0.7 proportionally increases with VBP. A recent study by Afifipour and Moarefvand (2014) utilized large diameter samples (100 mm and 150 mm) to study the UCS and Young's modulus of high volumetric block proportion (VBP = 70%–90%) bim-rocks. It is worthy to note the work of Kahraman and Alber (2006) who have studied, on laboratory size samples, the mechanical properties of an "inverse" bim-rock, with soft blocks and stiff matrix. They have reported an inverse trend for this bim-rock, decreasing UCS with increasing VBP, however this trend was found continuous over the range of VBP studied, 0.1 to 0.95. Elastic modulus was also found to decrease with VBP over the range of 0.2 to 0.7.

It should be noted that in studies reported herein, with the exception of Lindquist and Goodman (1994), the strength and elasticity modulus were determined in uniaxial compression. Moreover, radial strains were not measured and therefore Poisson's ratio was not determined. In practice the rock, and structures within, will be subjected to a tri-axial stress field. Moreover, an assumption of rock isotropy cannot be assumed without tri-axial mechanical testing.

An alternative approach to physical modeling is the use of numerical tools such as the finite elements method to calculate the relevant mechanical properties of bim-rocks using the basic properties of its block and matrix constituents. Xu et al. (2008) combined digital image processing and finite element modeling (FE) to simulate the large shear tests of SRM. The FE modeling also provides the capability to study the elastic properties and their variation in the sample. Furthermore it allows analyzing the macro-structure, i.e. engineering scale analysis, based on the microstructure of the material. Numerous publications were dedicated to the verification and validation of this approach, see Zohdi and Wriggers (2005); Geers et al. (2010) and references therein. The multi-scale approaches were also adopted and validated for the elastic analysis of concrete, an artificial bim-rock (e.g. Gal and Kryvoruk, 2011; Häfner et al., 2006; Wriggers and Moftah, 2006).

In this paper we present a parametric study of the elastic properties of bim-rocks using the theory of homogenization. We adopt the FE implementation (Gal et al., 2008), of the theory of homogenization, in which the heterogeneous material is replaced by an equivalent homogeneous continuum to develop a unit-cell for bim-rocks. We parametrically study the variation of the elastic moduli as function of VBP, block geometry and block–matrix stiffness ratio, focusing on the deviation from the initial assumption of isotropy. The results of the parametric study are compared with results of closed form solution (Hashin–Shtrikman) for multi-phase elastic solids. We then continue with an example of the multi-scale homogenization approach: estimating the elastic constants of the Zin conglomerate.

2. Methods

2.1. Asymptotic theory of homogenization

The method for obtaining the macroscopic behavior of a material, based on its microstructure is referred to as the theory of homogenization, by which the heterogeneous material is replaced by an equivalent homogeneous continuum. The method is performed on a statistically representative sample of material, referred to as a material unit cell. Numerous theories have been developed to predict the behavior of composite materials. Starting from the various effective properties obtained by the models of Eshelby (1957); Hashin (1962); Mori and Tanaka (1973), the self-consistent approaches of Hill (1965) and various mathematical homogenization methods (e.g. Christensen, 1979) pioneered by Benssousan et al. (1978) and Sanchez-Palencia (1980). Unfortunately, most of these analytical models can only give estimates or boundaries for the macroscopic properties, and the simplifying assumptions used, result in, the major differences obtained. Computational procedures for implementing homogenization have been an active area of research starting with the contribution by Guedes and Kikuchi (1990) for linear elasticity problems. Over the past two decades major contributions have been made to extend the theory of computational homogenization and improving fidelity and computational efficiency of numerical simulations, refer to Gal and Krivoruk (2011) and references therein. These developments established the finite element method (FEM) as one of the most efficient numerical methods, whereby the macroscopic responses can be obtained by volumetrically averaging numerical solutions of unit cells (e.g. Zohdi and Wriggers, 2005).

The asymptotic theory of homogenization is based on the following asymptotic expansion of the displacement field:

$$u(\mathbf{x},\mathbf{y}) = u^0(\mathbf{x},\mathbf{y}) + \xi^1 u^1(\mathbf{x},\mathbf{y}) + \xi^2 u^2(\mathbf{x},\mathbf{y}) + O(h) \quad (1)$$

where $\mathbf{x}$ and $\mathbf{y}$ are the position vectors in macroscopic and microscopic scales respectively. The scales are related through a scale factor $0 < \xi \ll 1$, such that $x = y/\xi$.

Assuming

$$u^0(\mathbf{x},\mathbf{y}) = u(\mathbf{x}) \quad (2)$$

$$u^1(\mathbf{x},\mathbf{y}) = \varepsilon^0(\mathbf{x})\chi_{imn}(\mathbf{y}) \quad (3)$$

where $\varepsilon^0(\mathbf{x})$ is the macro-scale strain vector and $\chi_{imn}(\mathbf{y})$ is the micro-scale influence function, we obtain two coupled problems.