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Quantifying stratigraphic uncertainties by stochastic simulation techniques based on Markov random field

Zhao Li [a], Xiangrong Wang [b], Hui Wang [c], Robert Y. Liang [a,*]

[a] Department of Civil Engineering, The University of Akron, Akron, OH 44325-3905, USA
[b] College of Engineering, Peking University, Beijing 100871, China
[c] Graduate School AICES, RWTH Aachen University, Schinkelstraße 2, Aachen, 52062, Germany

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ABSTRACT

Stratigraphic (or lithological) uncertainty refers to the uncertainty of boundaries between different soil layers and lithological units, which has received increasing attention in geotechnical engineering. In this paper, an effective stochastic geological modeling framework is proposed based on Markov random field theory, which is conditional on site investigation data, such as observations of soil types from ground surface, borehole logs, and strata orientation from geophysical tests. The proposed modeling method is capable of accounting for the inherent heterogeneous and anisotropic characteristics of geological structure. In this method, two modeling approaches are introduced to simulate subsurface geological structures to accommodate different confidence levels on geological structure type (i.e., layered vs. others). The sensitivity analysis for two modeling approaches is conducted to reveal the influence of mesh density and the model parameter on the simulation results. Illustrative examples using borehole data are presented to elucidate the ability to quantify the geological structure uncertainty. Furthermore, the applicability of two modeling approaches and the behavior of the proposed model under different model parameters are discussed in detail. Finally, Bayesian inferential framework is introduced to allow for the estimation of the posterior distribution of model parameter, when additional or subsequent borehole information becomes available. Practical guidance of using the proposed stochastic geological modeling technique for engineering practice is given.

1. Introduction

In recent years, the uncertainty arising from characterizing inherently heterogeneous soil median has received increasing attention in geotechnical engineering. Soil heterogeneity can be attributed to two main sources (Elkateb et al., 2003). The first source of heterogeneity, which can be called the inherent spatial variation of soil properties, is that within a single formation layer, the soil properties are different from one point to another in space due to the difference of geological deposition history and human activities. The second source of heterogeneity is the stratigraphic or lithological uncertainty, which can be interpreted as the uncertainty of interfaces (boundaries) between different soil layers or lithological units due to limited subsurface investigation data. Substantial research work has been performed on the former type of soil heterogeneity within one nominally homogeneous layer by using either geo-statistics or random field theory. The common practice is to apply Gaussian random field equipped with specific correlation structure to simulate the spatial variability of soil properties with consideration of inherent spatial correlation (Fenton, 1999; Griffiths and Fenton, 2004; Zhu and Zhang, 2013). For this modeling framework, recent studies have been reported to characterize the soil parameters by using site exploration data (Cao and Wang, 2012; Gong et al., 2014; Wang and Cao, 2013). However, the treatment of this type of uncertainties resulting from lithological heterogeneity has been dealt with mainly by using engineering judgment based on local experience (Elkateb et al., 2003). In contrast, the work presented in this paper focuses on stochastic modeling techniques for quantifying uncertainties of geological structure due to limited site exploration data.

Modeling soil profiles for a project site is commonly done by interpolation using a set of observations from borehole logs spaced some distance apart coupled with local geological experience (Nobre and Sykes, 1992). For such purpose, several approaches based on geo-statistical methods or interpolation methods have been established (Auerbach and Schaeben, 1990; Blanchin and Chilès, 1993; Chilès et al., 2004). However, most of these methods focus on estimate of the subsurface structure (geological model) with the *Maximum Probability Estimate* (MPE) conditional on local experiences, e.g. kriging, which is point estimation, and hence the lithological uncertainty cannot be quantified. Recently, probabilistic approaches have been developed to determine underground soil stratification based on cone penetration test (CPT) data (Cao and Wang, 2012; Ching et al., 2015; Wang et al., 2013) and to identify soil strata in

* Corresponding author.
E-mail addresses: zl28@zips.uakron.edu (Z. Li), superwxr@pku.edu.cn (X. Wang), wang@aices.rwth-aachen.de (H. Wang), rliang@uakron.edu (R.Y. Liang).

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London Clay formation based on water content data (Wang et al., 2014). Among these studies, the contact points of the subsurface stratigraphy are determined probabilistically. In the work presented herein, the spatial correlation of lithological units is defined by a graphical model (*neighborhood system*). Multiple stochastic realizations of subsurface configuration are generated. The statistical analysis of the realizations is used subsequently for stratigraphic uncertainty quantification.

In the context of stochastic simulation of subsurface structure, the Markov chain modeling method (Elfeki and Dekking, 2001) and multiple-point geo-statistics (Caers and Zhang, 2004) are capable of generating multiple realizations. However, these techniques may suffer certain limitations, such as stationary assumption, or that only a single type of observation data is used (i.e. borehole logs). To further relax these potential limitations, we have developed an innovative simulation method based on Markov random field. This method aims to reflect the inherent heterogeneous (multiple geo-material types), anisotropic (directionally dependent local correlation) and non-stationary (local correlation differs among different points in space) characteristics of geological body, as well as taking into consideration of the intrinsic local correlation of geological structure. Three types of site investigation data could be used as input in this model, including ground surface soil types, boundaries of different soil layers at each borehole log location, and strata orientation information (e.g., from ground penetration radar test and/or seismic survey data). Two stochastic modeling techniques are developed to generate the corresponding subsurface lithological unit configurations with the purpose of accommodating different confidence levels on geological structure type (i.e., how much confidence do we have regarding prior information showing layered structure or not). The concept of “information entropy” originally suggested by Wellmann and Regenauer-Lieb (2012) for a quantitative measure of uncertainty in geological modeling, is adopted herein to quantify stratigraphic uncertainty in the post-processing stage.

This paper is organized as follows. In Section 2, the proposed stochastic geological model is introduced, including model framework and stochastic simulation process. In Section 3, two modeling approaches, together with the corresponding numerical example results and model parameter sensitivity analysis are presented. To illustrate the performance of these two modeling approaches, the sensitivity analysis is carried out. In particular, the influence of discretizing mesh density and model parameter on the simulation results is systematically studied. In Section 4, the Bayesian inferential framework is introduced to allow for estimating model parameter for several scenarios when additional borehole data may subsequently become available for updating purpose. Finally, conclusions are presented in Section 5.

2. Stochastic geological model

The geological body is considered as a spatially correlated system with a certain configuration of different lithological units (i.e., geo-material layers and layers' orientation information). Markov random field theory as one of the most sophisticated spatial statistical models provides a convenient and consistent way for modeling context dependent entities such as correlated features and for analyzing the spatial dependencies of physical phenomena (Li, 2009; Zhang et al., 2001). In this study, the naturally occurring geological body is assumed to be in a “stable state” and the intrinsic spatial correlation of geological structure can be modeled by *contextual constraint* using MRF theory. The root of such assumption is based on its successful application in geo-statistics, i.e., MRF has been used to model discrete geological structures (Norberg et al., 2002), and to consider geological realism and connectivity (Daly, 2005) as well as in geological mapping (Tolpekin and Stein, 2009). Meanwhile, as proven by the Hammersley–Clifford theorem (Besag, 1974; Hammersley and Clifford, 1971), a MRF process is equivalent to a given Gibbs random field. Such MRF-Gibbs equivalence makes it possible to represent the joint probability distribution of MRF in an explicit formula of energy function (Geman and Geman, 1984), thereby provides a feasible and powerful mechanism for modeling spatial continuity and aggregation of the stratigraphic profile.

2.1. Neighborhood system, Markov random field and Gibbs distribution

2.1.1. Neighborhood system

The proposed geological model is constructed by discretizing the geological body of interest into small square elements. A *neighborhood system* is developed to represent the spatial correlation.

Let $\mathbf{S}=\{i|i=1,2,\ldots,n\}$ be the set of elements in which i is an element index. In an MRF, the elements in $\mathbf{S}$ are related to one another via a *neighborhood system*, which is defined as $\mathbf{N}=\{\mathbf{N}_i|i\in\mathbf{S}\}$. $\mathbf{N}_i$ is the set of all elements which share common node(s) with element i in the meshed plot. Fig. 1 shows an example of a local *neighborhood system*. Element i has a local *neighborhood system* $\mathbf{N}_i$ containing 8 neighbors $\{j_1,\ldots,j_8\}$ but not including itself, and the neighboring relationship is mutual. Note that boundary element has fewer neighbors.

2.1.2. Markov random field and Gibbs distribution

Let $\mathbf{R}=\{R_i,i\in\mathbf{S}\}$ be a set of random variables indexed by $\mathbf{S}$, in which each random variable R_i takes a label r_i (i.e., lithological unit label, such as sand, clay, shale, etc.) in its state space $\mathbf{L}=\{1,2,\ldots,m,\ldots l\}$ of all lithological units (or labels). The event $\mathbf{R}=\mathbf{r}$ indicates the joint event $(R_i=r_i,i\in\mathbf{S})$, where $\mathbf{r}=\{r_1,\ldots,r_n\}$ denotes a subsurface configuration of $\mathbf{R}$, corresponding to a realization of this random field $\mathbf{R}$. Let $\mathfrak{R}=\{\mathbf{r}=$

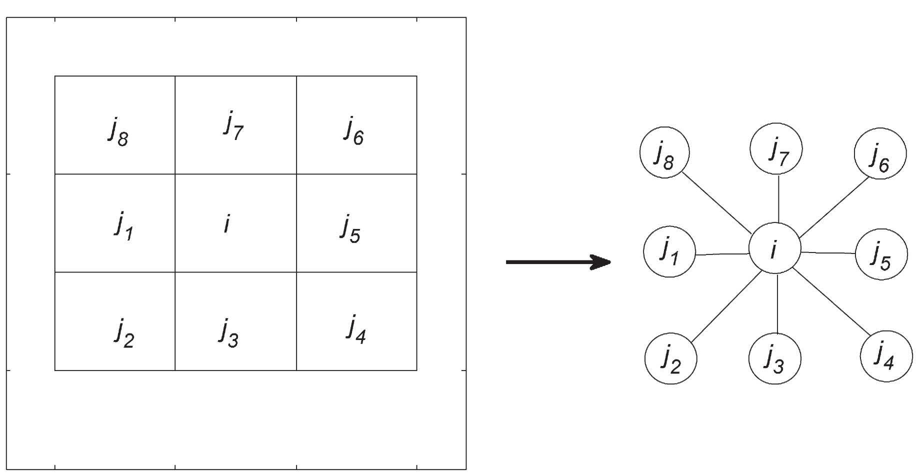

Fig. 1. Local neighborhood system.