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Estimation of deformation modulus of rock masses based on Bayesian model selection and Bayesian updating approach

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ABSTRACT

The deformation modulus is one of the most important parameters to model the behavior of rock masses, but its direct measurement by in situ tests is costly, time-consuming and sometimes infeasible. For that reason, many models have been proposed to estimate the deformation moduli of rock masses based on geotechnical classification indices, such as the Rock Mass Rating (*RMR*), the Geological Strength Index (*GSI*), the Tunneling Quality Index (*Q*), or the Rock Quality Designation (*RQD*). We present an approach, based on model selection criteria —such as Akaike information criterion (*AIC*), Bayesian information criterion (*BIC*) and deviance information criterion (*DIC*)— to select the most appropriate model, among a set of four candidate models —linear, power, exponential and logistic—, to estimate the deformation modulus of a rock mass, given a set of observed data. Once the most appropriate model is selected, a Bayesian framework is employed to develop predictive distributions of the deformation moduli of rock masses, and to update them with new project-specific data that significantly reduce the associated predictive uncertainty. Such Bayesian updating approach can, therefore, affect our computed estimates of probability of failure, which is of significant interest to reliability-based rock engineering design.

1. Introduction

The deformation modulus of a rock mass is one of the most important parameters influencing its mechanical behavior (Kayabasi et al., 2003). Therefore, it is one of the most commonly required input parameters for both analytical and numerical methods in geotechnical engineering, and it is the basis for many geotechnical analyses (Palmström and Singh, 2001).

According to the Commission of Terminology of the International Society of Rock Mechanics (ISRM, 1975), the deformation modulus of a rock mass, E_{rm}, is defined as "the ratio of stress to corresponding strain during loading of a rock mass including elastic and inelastic behavior". There are many in situ tests to directly measure E_{rm} such as plate loading or plate jacking tests, dilatometer tests, flat jack tests, pressure chamber tests, etc. (Bieniawski, 1978; Palmström and Singh, 2001; Hoek and Diederichs, 2006; Kang et al., 2012). However, such in situ tests are time consuming, expensive and sometimes even infeasible. For that reason, many models have been proposed to indirectly estimate the deformation moduli of rock masses based on geotechnical classification indices such as the Rock Mass Rating (*RMR*), the Geological Strength Index (*GSI*), the Tunneling Quality Index (*Q*), or the Rock Quality Designation (*RQD*) (Bieniawski, 1978; Serafim and Pereira, 1983; Nicholson and Bieniawski, 1990; Barton, 1996; Sonmez et al., 2004; Hoek and Diederichs, 2006; Zhang, 2010). The most commonly used models are summarized in Table 1.

Since dozens of models (see Table 1) have been proposed during the past decades, engineering practitioners are often concerned about "how to choose the most appropriate model" to be used in a particular project for which there is a set of available data. To tackle this problem, a suitable model selection method should be adopted (see e.g., Burnham and Anderson (2002) for details). Since most of the models (i.e., empirical correlations) listed in Table 1 are obtained using regression methods, one may naturally think that comparing the R^2 of different models is the most convenient approach for model selection. But this approach only measures the goodness of fit of the model; model complexity is ignored, hence always favoring "fuller" models with more parameters. In other words, "neglecting the principle of *parsimony* makes it a poor technique for model selection" (Johnson and Omland, 2004). In contrast, model selection criteria based on "information criteria" consider both model fit and complexity, hence being more suitable for model selection. While model selection criteria have been previously used in geotechnical engineering (Honjo et al., 1994; Li et al., 2013; Tang et al., 2013a, 2013b; Cao and Wang, 2013; Wang et al., 2013; Cao and Wang, 2014a, 2014b; Wang et al., 2014), we hope that the application presented herein can help, in conjunction with other recent contributions (Wang and Aladejare, 2015), to promote a wider use of model selection in rock engineering and, in

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Table 1
A summary of models to estimate the deformation moduli of rock masses based on geotechnical classification indices.

Proposed by	Empirical correlations	Input parameters	Types	Limitations
Bieniawski (1978)	$E_{rm} = 2RMR - 100$	RMR	Linear	$RMR > 50$
Serafim and Pereira (1983)	$E_{rm} = 10^{((RMR - 10) / 40)}$	RMR	Exponential	
Kim (1993)	$E_{rm} = 300 \times 10^{-3}\exp(0.07RMR)$	RMR	Exponential	
Read et al. (1999)	$E_{rm} = 0.1(RMR / 10)^3$	RMR	Power	
Jašarević and Kovačević (1996)	$E_{rm} = \exp.(4.407 + 0.081 * RMR)$	RMR	Exponential	
Aydan et al. (1997)	$E_{rm} = 0.0097RMR^{3.54}$	RMR	Power	
Verman et al. (1997)	$E_{rm} = 0.4H^{\alpha}10^{((RMR\text{-}20)/38}$	RMR, H, α	Exponential	
Diederichs and Kaiser (1999)	$E_{rm} = 7(\pm 3)\sqrt{10^{(RMR-44)/21}}$	RMR	Non-linear	
Galera et al. (2005)	$E_{rm} = 0.0876RMR$	RMR	Linear	$RMR > 50$
Galera et al. (2005)	$E_{rm} = 0.0876RMR + 1.056(RMR - 50) + 0.015\,(RMR - 50)^2$	RMR	Polynomial	$RMR \le 50$
Shen et al. (2012)	$E_{rm} = 100\exp[-((RMR - 100) / 37)^2]$	RMR	Gaussian function	
Khabbazi et al. (2013)	$E_{rm} = 9 \times 10^{-7}RMR^{3.868}$	RMR	Power	
Nicholson and Bieniawski (1990)	$E_{rm} = E_i(0.0028RMR^2 + 0.9\exp(RMR / 22.82))$	RMR, E_i	Non-linear	
Mitri et al. (1994)	$E_{rm} = E_i[0.5(1 - \cos(\pi RMR / 100))]$	RMR, E_i	Trigonometric	
Aydan and Kawamoto (2000)	$E_{rm} = E_i[RMR / (RMR + \beta(100 - RMR))]$	RMR, E_i, β	Fractional	
Galera et al. (2005)	$E_{rm} = E_i \exp.((RMR - 100) / 36)$	RMR, E_i	Exponential	
Sonmez et al. (2006)	$E_{rm} = E_i 10^{[(RMR - 100)(100 - RMR) / 4000 * \exp.(-RMR / 100)]}$	RMR, E_i	Exponential	
Shen et al. (2012)	$E_{rm} = E_i \exp.[-((RMR - 116) / 41)^2]$	RMR, E_i	Gaussian function	
Hoek and Brown (1997)	$E_{rm} = \sqrt{\frac{\sigma_{ci}}{100}}10^{((\mathrm{GSI} - 10) / 40)}$	GSI, σ_{ci}	Exponential	
Hoek et al. (2002)	$E_{rm} = (1 - D / 2)\sqrt{\frac{\sigma_{ci}}{100}}10^{((\mathrm{GSI} - 10) / 40)}$	GSI, σ_{ci}, D	Exponential	$\sigma_{ci} < 100$
Hoek et al. (2002)	$E_{rm} = (1 - D/2)\ 10^{((\mathrm{GSI}-10)/40)}$	GSI, D	Exponential	$\sigma_{ci} > 100$
Hoek and Diederichs (2006)	$E_{rm} = E_i\left[0.02 + \frac{1-D/2}{1+e^{\frac{60+15D-GSI}{11}}}\right]$	GSI, E_i, D	Sigmoid equation	
Hoek and Diederichs (2006)	$E_{rm} = 100\left[\frac{1-D/2}{1+e^{\frac{75+25D-GSI}{11}}}\right]$	GSI, D	Sigmoid equation	
Sonmez et al. (2004)	$E_{rm} = E_i\,(s^a)^{0.4}$; $s = \exp.[(GSI - 100) / (9 - 3D)]$, $a = 0.5 + 1 / 6(\exp(-GSI / 15) + \exp.(-20 / 3))$	GSI, E_i, D	Non-linear	
Barton (1983)	$E_{rm} = 25\log_{10}(Q)$	Q	Logarithmic	$Q > 1$
Barton (1995)	$E_{rm} = 10Q^{1/3}$	Q	Power	
Palmström and Singh (2001)	$E_{rm} = 8Q^{0.4}$	Q	Power	$1 < Q < 30$
Kang et al. (2012)	$E_{rm} = 10^{(0.32\log Q + 0.585)}$	Q	Exponential	
Barton (1996)	$E_{rm} = 10Q1/3\ \mathrm{c}$; $Q_c = Q(\sigma_{ci} / 100)$	Q, σ_{ci}	Power	
Coon and Merritt (1970)	$E_{rm} = E_i(0.0231RQD - 1.32)$	RQD, E_i	Linear	
Kayabasi et al. (2003)	$E_{rm} = 0.135[E_{\mathrm{i}}\,(1 + RQD / 100) / WD]^{1.1811}$	RQD, E_i, WD	Non-linear	
Gokceoglu et al. (2003)	$E_{rm} = 0.001[(E_{\mathrm{i}}/\sigma_{\mathrm{ci}})\,(1 + RQD / 100) / WD]^{1.5528}$	RQD, E_i, σ_{ci}, WD	Non-linear	
Zhang and Einstein (2004)	$E_{rm} = E_i\ 10^{(0.0186RQD - 1.91)}$	RQD, E_i	Exponential	
Palmström and Singh (2001)	$E_{rm} = 7RMi^{0.4}$	RMi	Power	

E_{rm} deformation modulus of rock mass (GPa), H depth of tunnel (m), α a parameter related to RMR, σ_{ci} uniaxial compressive strength of intact rock (MPa), E_i elastic modulus of intact rock (GPa), WD weathering degree, β a constant to be determined using a minimization procedure for experimental values, RMi rock mass index.

particular, in application related to the estimation of the deformation modulus of rock masses.

Model selection criteria can be used to identify the "best" model, although predictions conducted with such "best" models are sometimes associated to large uncertainties due to the high variability of data employed to build them. Within this context, further improvements could be achieved with a Bayesian updating approach, so that uncertainties can be reduced when new project-specific observations become available, hence helping designers to "maximize the value of new project-specific observations" (Zhang et al., 2004). This is because the Bayesian updating is expected to reduce the predictive uncertainty (i.e., the COV of the predictive distribution), which may be of interest to reliability-based designs, as the computed probability of failure of a design will be influenced by such predictive uncertainty.

In this study, we present an approach to select the most appropriate model, among four commonly used candidate models, to estimate the deformation modulus of a rock mass, E_{rm}, based on its RMR or GSI values. The proposed approach builds on the use of "information criteria" to rank models, considering both their fit and complexity. In addition, we propose a procedure, within the Bayesian framework for model updating, to systematically update the predictive distribution of E_{rm}, and its associated predictive uncertainty, when new "project-specific" data are available. Finally, we use an example case of an assumed circular rock tunnel design to show that the reduction of predictive uncertainties achieved with the Bayesian updating has a significant influence on the computed reliability estimates of a rock engineering design.

2. Bayesian models: inference, selection and prediction

2.1. Introduction

The relationship between observed (input or independent) and estimated (output or dependent) variables in real rock engineering problems, such as estimating the deformation modulus of rock masses, is often complex. But, in some cases, we may build a model, $z_i = M_i(\mathbf{x}, \mathbf{\Theta})$, to describe such complex behavior, where z_i $(i = 1, 2, \ldots, r)$ denote the (observable) dependent variables, $\mathbf{x} = (x_1, x_2, \ldots, x_m)$ is the m-dimensional vector of the (observable) independent variables, and $\mathbf{\Theta} = (\theta_1, \theta_2, \ldots, \theta_p)$ is the p-dimensional vector of (unobservable) model parameters that need to be estimated based on observed data. (Geyskens et al., 1998). $M_i(\cdot,\cdot)$ represents the relationship between them. For instance, the correlation proposed by Serafim and Pereira (1983) to estimate the deformation modulus of a rock mass, i.e., $E_{rm} = 10^{(a*RMR+b)}$, is a model with $r = 1$, in which the input variable $\mathbf{x} = RMR$ is one dimensional $(m = 1)$, and in which the vector of parameters is given by $\mathbf{\Theta} = (a = 1/40, b = -1/4)$ (Note, therefore, that $p = 2$). That is, in this model, RMR is the (observable) independent variable, E_{rm} is the (observable) dependent variable, and $\mathbf{\Theta} = (a, b)$ are the (unobservable) model parameters to be estimated. Since a model is only an approximation to the complex real-world phenomenon, models are usually imperfect and their predictions are uncertain (Geyskens et al., 1993). In addition, simplifications are unavoidable and, as it is clear from Table 1, there may be many different models for the same task. Here, we use a Bayesian approach to infer and update a set of model parameters, $\mathbf{\Theta}$, for