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Technical note

Effects of nonlinear failure criterion on the three-dimensional stability analysis of uniform slopes

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ABSTRACT

This paper presents a kinematic approach of limit analysis to evaluate the effects of nonlinear failure criterion on the three-dimensional (3D) stability analysis of uniform slopes. For slopes in various clays, the upper bounds on the critical heights associated with the linear and nonlinear failure criteria are obtained and comparisons of the linear and nonlinear results are made to investigate their differences. The differences between the linear and nonlinear solutions change with varying slope inclination, resulting in two critical values on the slope inclination angle. In the intermediate range of slope inclinations, the critical heights associated with the linear failure criterion are smaller than the nonlinear results, and the underestimation by using the linear failure criterion cannot be neglected. For other cases, the analyses based on the linear failure criterion will overestimate the stability of actual slopes obeying the nonlinear failure criterion. Meanwhile, the 3D effects have insignificant influences on the two critical slope inclination angles. However, the sensitivity of the critical heights to the nonlinearity of soil strength envelope appears to be more pronounced for slopes with a smaller width.

1. Introduction

In the past, most slope stability analyses were conducted using the linear Mohr–Coulomb (MC) failure criterion. However, many experimental results demonstrated that the failure envelopes for most soils are not linear, especially in the range of small normal stresses (e.g., Penman, 1953; Bishop et al., 1965; Ponce and Bell, 1971; Lefebvre, 1981; Atkinson and Farrar, 1985; Maksimovic, 1989). Hence, various nonlinear failure criteria were proposed to fit the curved failure envelopes of soils (e.g., De Mello, 1977; Lefebvre, 1981; Skempton, 1985; Zhang and Chen, 1987; Maksimovic, 1989; Baker, 2004a). These nonlinear criteria have been widely applied into the assessment of slope stability.

To investigate the effects of the nonlinear failure criteria on the assessment of soil slope stability, many attempts have been made to incorporate the nonlinear failure criteria into the conventional limit equilibrium method (e.g., Charles, 1982; Charles and Soares, 1984; Day and Axten, 1989; Perry, 1994; Srbulov, 1997; Popescu et al., 2000; Baker, 2003, 2004b, 2005; Jiang et al., 2003; Deng et al., 2014; Eid, 2014). Based on the kinematic approach of limit analysis, some researchers adopted a tangential technique to extrapolate the application of nonlinear failure criteria in the stability analysis of soil slopes (e.g., Zhang and Chen, 1987; Drescher and Christopoulos, 1988; Yang and Yin, 2004; Zhao et al., 2015). Furthermore, some other numerical methods have also applied the nonlinear failure criteria into the assessment of slope stability (e.g., Popescu et al., 2000; Li, 2007; Li and Cheng, 2012; Yang and Chi, 2013).

The previous results revealed that the slope stability analyses conducted by the linear MC strength envelope will lead to higher safety factors than those obtained by using the nonlinear failure criterion (e.g., Charles and Soares, 1984; Day and Axten, 1989; Srbulov, 1997; Popescu et al., 2000; Baker, 2003; Eid, 2014). Nevertheless, Baker (2004b) found that using the linear failure criterion could predict smaller critical heights rather than using the nonlinear criterion in the specific range of slope inclinations. Their results are limited to two-dimensional (2D) slope problems under plain-strain condition. As Jiang et al. (2003) presented, the results derived by the linear failure criterion may overestimate the stability of soil slopes with nonlinear failure criterion, especially for 3D problems. Therefore, this study will extend the kinematic approach of limit analysis for the stability of 2D slopes with nonlinear failure criterion into the 3D condition. Various soils with linear and nonlinear failure envelopes are adopted to obtain the upper bound solutions for uniform slope stability under plane-

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strain (2D) or 3D conditions. Some comparisons are made to investigate the 3D effects on the safety of slopes in soils obeying the nonlinear failure criterion.

2. Limit analysis of 3D slope stability

2.1. The nonlinear failure criterion and tangential technique

Zhang and Chen (1987) proposed a power-law type of nonlinear failure criterion for cohesive soils. Afterwards, this nonlinear criterion has been widely utilized in the slope stability analysis by many researchers (e.g., Zhang and Chen, 1987; Drescher and Christopoulos, 1988; Yang and Yin, 2004; Li, 2007; Deng et al., 2014). The power-law failure criterion can be expressed as follows:

$$\tau = c_0\left(1+\frac{\sigma_n}{\sigma_t}\right)^{1/m} \tag{1}$$

where τ and σ_n are shear and normal stresses on the failure surface, respectively. The parameter m is the nonlinearity coefficient, c_0 is the initial cohesion of soil at zero stress, and σ_t is the absolute value of tensile stress when τ is equal to zero, as shown in Fig. 1. The value of m must satisfy $m \geq 1.0$, and the values of c_0 and σ_t have to be larger than zero.

To incorporate the nonlinear failure criterion into the kinematic approach of limit analysis of slope stability, a tangential technique was proposed by Drescher and Christopoulos (1988) and then developed by Yang and Yin (2004). The nonlinear failure envelope is replaced by a simple tangential line at some point G in the form of the equivalent MC strength parameters, as shown in Fig. 1. The equation of tangential line at some point G can be expressed as

$$\tau = c' + \sigma_n \tan\phi' \tag{2}$$

where ϕ' is the equivalent friction angle; and c' is the equivalent cohesion, the intercept of the tangential line on the τ-axis. As Yang and Yin (2004) presented, the equivalent cohesion c' can be derived by the following expression:

$$\frac{c'}{c_0} = \frac{m-1}{m}\left[\frac{\sigma_t}{c_0} m \tan\phi'\right]^{\left(\frac{1}{1-m}\right)} + \frac{\sigma_t}{c_0}\tan\phi'. \tag{3}$$

For 2D plane-strain slope stability problems, the upper bound solutions obtained from the tangential technique were found to be in good agreement with the finite element solutions of Li (2007) and the numerical upper bound solutions of Li et al. (2008, 2009a). In addition, the study of Senent et al. (2013) indicated that the tangential technique is adequate for many cases of tunnel face stability applications. These results may verify the adequacy and accuracy of upper bound solutions derived from the tangential technique. This study will extrapolate the application of the tangential technique for the stability analysis of slopes from the 2D to the 3D condition.

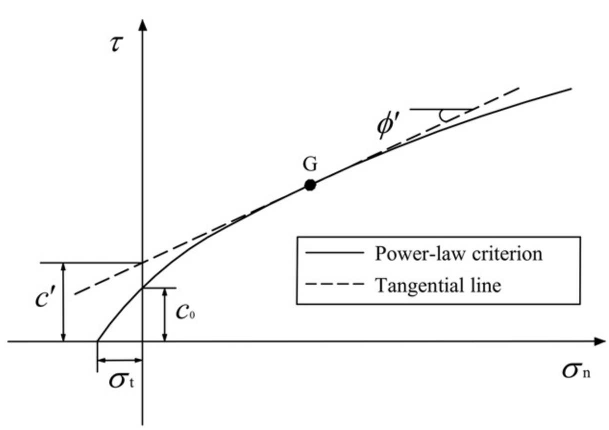

Fig. 1. The power-law failure criterion and its tangential line.

2.2. Kinematic approach of limit analysis with 3D failure mechanism

In the kinematic approach of limit analysis, Michalowski and Drescher (2009) proposed a 3D admissible rotational failure mechanism for slopes in soils obeying MC failure criterion, which was then extended by Zhang et al. (2013a) to involve the face failure and base failure. The results of Zhang et al. (2013b) demonstrated that the obtained upper bounds were more critical than the numerical upper bound solutions of Li et al. (2009b, 2010) and close to their lower bound solutions. It implies that the extended 3D failure mechanism can give the best estimate of the upper bound on the stability of slopes. Therefore, the extended 3D failure mechanism is adopted here to conduct the limit analysis of slope stability with a nonlinear failure criterion. As shown in Appendix A, Figs. A.1(a) and (b) illustrate the extended 3D mechanisms for face failure and base failure, respectively. Fig. A.2 shows the extended 3D failure mechanisms for slopes with finite width B, modified by a plane insert with the width b. For details of the construction of 3D admissible rotational failure mechanism, see the source reference.

Based on 3D failure mechanisms, the upper bound on the critical height H_{cr} can be determined by equating the rate of work W_γ done by soil weight to the rate of internal energy dissipation D. In general, the balance equation is given as follows:

$$W_\gamma^{\text{curve}} + W_\gamma^{\text{plane}} = D^{\text{curve}} + D^{\text{plane}} \tag{4}$$

The expressions of W_γ^{curve} and D^{curve} for the curvilinear cone at the two ends of the mechanism have been given in Appendix A for 3D face-failure and 3D base-failure mechanisms, respectively. The work rates of W_γ^{plane} and D^{plane} for the plane insert in the center of the mechanism can be found in the reference of Chen (1975). It should be noted that the parameters c and ϕ in their expressions should be replaced by c' and ϕ'.

In order to obtain the least upper bound on the critical height H_{cr}, an optimization method proposed by Chen (1992) is used in this study. Given a slope with certain values (i.e., slope inclination angle β, nonlinear parameters m, c_0, σ_t, and the ratio of width to height B/H), the critical height H_{cr} can be obtained with respect to independent variables: angles θ_0 and θ_h, ratio of r_0'/r_0, relative width of the plane insert b/H, ratio $n = H'/H$ for the 3D face-failure mechanism or angle β' for the 3D base-failure mechanism, and one additional variable ϕ'. Here, the variables θ_0, θ_h, H', and β' are illustrated in Fig. A.1; the variable r_0' relates to "OA′" and r_0 denotes "OA" in Fig. A.1. More detail for the optimization procedure can be found in Zhang et al. (2013a).

3. Results and discussions

3.1. Effects of nonlinear failure criterion on critical heights of 2D slopes

Based on the 2D plane-strain limit analysis presented by Yang and Yin (2004) and Chen (1975), the effects of the nonlinearity of soil strength envelope on the critical height H_{cr} for earth slopes are further investigated in this section. The differences between the critical heights associated with linear and nonlinear failure criteria are presented for homogeneous slopes in different kinds of cohesive soils.

From the literatures, four typical kinds of cohesive soils including Israeli clay, London clay, Upper Lias clay and Oxford clay are used here. For Israeli clay and London clay, this study utilizes the experimental data and corresponding linear strength envelopes reported by Baker (2003). The test data and linear strength parameters for Upper Lias clay and Oxford clay are adopted from Eid (2014) and Perry (1994),