Engineering Geology 196 (2015) 293–304

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

Modeling of minimum void ratio for sand–silt mixtures

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ARTICLE INFO

Article history:
Received 28 January 2015
Received in revised form 11 July 2015
Accepted 16 July 2015
Available online 30 July 2015

Keywords:
Void ratio
Packing density
Soil mixture

ABSTRACT

Minimum void ratio or maximum packing density is an important soil property in geotechnical engineering. It correlates to the volume change tendency, the pore fluid conductivity, and the shear strength of the soil. In geotechnical engineering, it often requires to estimate the minimum void ratio for a sand–silt mixture with any amount of fines content, based only on few laboratory test results. The minimum void ratio for soil mixtures is usually estimated by methods based on, to some extent, an empirical approach, for example, the AASHTO coarse particle correction method. In this paper, based on a more fundamental approach using the concept of dominant particle network, we aim to develop a mathematical model that can predict the minimum void ratio for sand–silt mixtures with any amount of fines content. The developed model only requires two parameters for the prediction of minimum void ratios of soil mixtures with various fines contents. The developed model is evaluated by the experimental results on 33 types of soil mixtures available in the literature, including mixtures of sands (Ottawa sand, Nevada sand, Toyoura sand, Hokksund sand, etc), and silts (ATC silt, Nevada fines, crushed silica fines, grind Toyoura fines, etc). Comparisons of the results are discussed.

1. Introduction

Granular soil is a packing of soil particles of different sizes. Research on soil mechanics, for several decades, revealed that the amount of fines in a sand–silt mixture has significant effects on its mechanical properties (e.g. Selig and Ladd, 1973; Aberg, 1992; Miura et al., 1997; Cubrinovski and Ishihara, 2002; Bobei et al., 2009; Peters and Berney, 2010; Fuggle et al., 2014). This is not surprising because how particles are packed is greatly influenced by the particle size distribution, which is an important factor governing the properties of materials. The importance of particle size distribution has also been observed in many branches of industry, such as ceramic processing (Reed, 1995), powder metallurgy (Smith, 2003), and concrete mixes (Powers, 1968).

Studies of packing density as a function of particle size distribution were meager published around 1930s. Research interest of high-density packing of ceramics and metal particles was renewed around 1954, for the reason of impetus of atomic energy and space research. However, the research works were mainly considering packing of uranium oxide and optimum particle size distribution (PSD) for maximum packing density (McGeary, 1961). For soils, a method of prediction of maximum packing density of soil with different sizes of particles was proposed by Humphres (1957) using an empirical and graphical method. Around 1986, AASHTO T 224-86 specifications postulate an empirical method for estimating the maximum packing density by using a "correction factor" for coarse particles that can be applied when the percent of gravel size particles is less than or equal to 70%. Kezdi (1979) outlined an analytical method to estimate the minimum porosity of a binary mixture of granular soils. The method is based on the ideal situation that the pore space among large particles is fully filled by the fine particles without alternating the packing structure of large particles. Thus, the method is applicable only to very small size of fine particles and often overestimates the maximum packing density. For improving compaction control of granular fill, Fragaszy and Sneider (1991) carried out an extensive set of experiments on soils with a wide range of particle sizes, and compared the measured maximum dry density with the two empirically based predictive methods: "Humphres method (Humphres, 1957)" and "AASHTO correction factor" method (AASHTO, 1986). In association with the liquefaction potential of silty-sand, Lade et al. (1998) had carried out minimum void ratio tests for different types of soil mixtures. They also proposed an analytical method for predicting the minimum void ratio for spheres with different sizes; however, this method is applicable only to an ideal situation that the small particles are much smaller than the large ones. Vallejo (2001) measured porosities on mixtures of two different sizes of glass beads. He also proposed an equation with similar form to the method by Kezdi (1979) for estimating the porosity of the binary mixtures. He indicated that the theoretical minimum porosity was very difficult to achieve in laboratory mixtures. Cubrinovski and Ishihara (2002) examined a large number of test data on silty-sand and presented a set of empirical equations to show the influence of fines content on the magnitude of minimum void ratio. Apart from these studies, computer simulation analyses using discrete element method have also been

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http://dx.doi.org/10.1016/j.enggeo.2015.07.015
0013-7952/

implemented to study the characteristics of the void ratio of particle mixtures (An, 2013; Fuggle et al., 2014). The trend of computer simulation results resembles that obtained from experimental tests. Nevertheless, these methods are not yet capable of predicting the minimum void ratio for sand–silt mixtures.

A more extensive research on analytical method has been carried out in the field of concrete mixes by de Larrard (1999) that can be used to predict packing density of concrete mixes of aggregate and sand. This method has been widely used for concrete mixture design to optimize the packing densities of cement, mortar and concrete (e.g. Kwan and Fung, 2009; Fennis et al., 2013). Methods similar to the formulation by de Larrard (1999) can also be found in the field of powder mixes by Stovall et al. (1986) and Yu and Standish (1987), which are commonly used in the pharmaceutical industry.

However, the applicability of these existing analytical methods (similar to that given by de Larrard, 1999) has not yet been examined for the packing density of sand–silt mixtures with different particle sizes. In this study, the existing packing model by de Larrard (1999) is evaluated by comparing the measured and predicted minimum void ratios for a number of silt–sand mixtures. Deficiencies of the existing packing models are identified, and a new model is proposed that can better predict the minimum void ratios for sand–silt mixtures with different particle sizes.

2. Existing packing theories and models

The minimum void ratio is 0.35 for a hexagonal packing of monosize spheres. The minimum void ratio for a randomly arranged packing is about 0.56–0.66. The particle shape has noteworthy influence on the value of minimum void ratio, which is generally lower for more spherical particles and higher for less spherical (or more angular) particles. When it comes to a packing of particles with different sizes, the minimum void ratio is also governed by the particle size distribution. Considering the simplest case of a binary mixture of particles with two sizes, the experimental results on steel shot mixtures given by McGeary (1961) are illustrated in Fig. 1. The packing density is plotted for large particles of 3.14 mm mixed with six other sizes (i.e., 0.91, 0.66, 0.48, 0.28, 0.19, and 0.16 mm). This figure shows the characteristics of packing density change due to fines content.

When the fines content is low, the smaller particles would fill the voids among the larger particles and thereby increase the packing density. Upon an increase of fines content, the voids among the large particles are eventually fully occupied and thereby the maximum packing density is reached. As the fines content continues to increase, the reverse trend is observed (i.e., the packing density decreases). The decrease of packing density is due to the fact that large particles are pushed apart by the small particles. As the fines content increases further, eventually the volume of small particles becomes much greater than that of large particles, and the larger particles would present as isolated inclusions embedded within the network of the smaller particles. Hence, as shown in Fig. 1, mixing particles of two different sizes would in general have a greater packing density than packing with one particle size.

The experimental results in Fig. 1 also show that the relative size of the large and small particles is an important factor influencing the packing density. It is obvious that, in order for the small particles to be fit into the voids between large particles, the small particles should be relatively smaller than the large particles. For a packing of spheres, the size of small particles should be at least 6.5 times smaller of the large particle size in order to fit in the tetrahedral cavities of the sphere packing. The effect of relative particle size on the packing density was shown by McGear and replotted in Fig. 2 for fines content of 24%. The packing density increases (or the void ratio decreases) significantly for particle size ratio less than 7. Larger than this value, the packing efficiency decreases rapidly.

To cater for multiple mixes of different size particles, the above binary packing model has been extended to a variety of packing models, most of which are based on the linear packing theory (Westman and Hugill, 1930) and may thus be classified as linear packing models. The linear packing theory postulates that for the multiple components (each comprising of all the particles of a certain size) mixed together, the change of packing density is a linear combination of the two mechanisms: (1) the inserted small particles fill voids of the packing, and (2) the inserted large particles embedded in the matrix of the packing. In the early age theory, the particle size ratio was not considered. In the 1980s, this theory has been refined to account for the effect of particle size ratio by Stovall et al. (1986), Yu and Standish (1987), and de Larrard (1999).

The packing density equations proposed in the afore-mentioned packing density models have the same expression. The equation in terms of the notation given by de Larrard (1999) is as follows

$$\gamma_i = \frac{\beta_i}{1-\sum_{j=1}^{i-1}\left[1-\beta_i + w(r)\beta_i(1-1/\beta_j)\right]y_j - \sum_{j=i+1}^{n}\left[1-l(r)\beta_i/\beta_j\right]y_j} \tag{1}$$

where γ_i is the predicted packing density of a mixture consisting of n components. It requires the input of the packing density of each component and the solid volumetric fraction of each component (i.e. particle size distribution). Considering component i is dominant, β_i and β_j are the packing densities of components i and j, y_i is the solid volumetric fraction of component j, r is the size ratio between the components i and j, and $l(r)$ and $w(r)$ are the interaction functions accounting for the effects of particle size ratio. The two functions are termed as "loosening function" and "wall function", respectively.

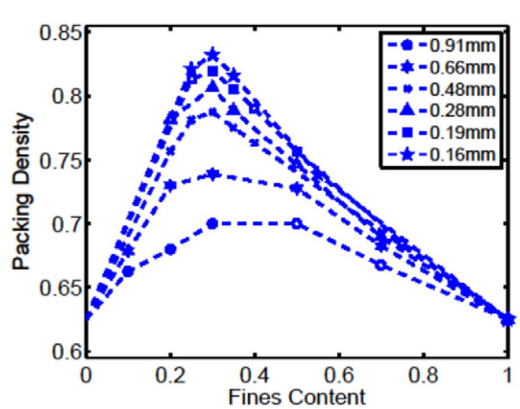

Fig. 1. Binary packing of steel shots. Data from McGeary (1961).

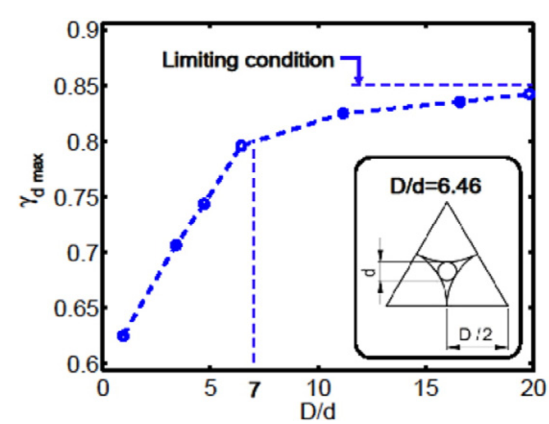

Fig. 2. Effect of particle size ratio on maximum packing density. Data from McGeary (1961).